

MATHEMATICAL MODEL OF PRESSURE AND TEMPERATURE DISTRIBUTION IN WORKING AREA OF MINE LOCOMOTIVE DISC BRAKE

МАТЕМАТИЧНА МОДЕЛЬ РОЗПОДІЛУ ТИСКУ І ТЕМПЕРАТУРИ В РОБОЧІЙ ЗОНІ ДИСКОВОГО ГАЛЬМА ШАХТНОГО ЛОКОМОТИВА

Purpose. Calculate the coordinates of the maximum temperature and the greatest pressure on the working surface for the rational parameters of the main elements of the disc brake of the mine locomotive with a multi-sector brake disc selected by mathematical modeling.

The methods. To find the coordinates of the maximum temperature and the greatest pressure on the working surface of a disc brake with a multi-sector brake disc with selected rational parameters, mathematical modeling of temperature and pressure on the friction surface was carried out.

Findings. On the basis of mathematical modeling, the maximum temperature and its coordinates and the greatest pressure on the working surface of a disc brake with a multi-sector brake disc were found. It is shown that the maximum temperature on the friction surface of the main elements of the disc brake with the selected parameters in specific mine conditions under the most unfavorable operating conditions will not exceed the permissible value.

The originality. A mathematical model of braking of a mine locomotive with a disc brake was developed, which creates a pulsating braking moment on the axle of the wheel pair, which depends on its angular coordinate, taking into account the non-linear dependence of the coupling coefficient on the relative slip, on the basis of which the parameters of the braking moment are established, which allow to improve the braking characteristics.

Practical implementation. A scientifically based engineering methodology for choosing rational parameters of the disc brake of a mine locomotive and determining the dynamic and kinematic characteristics of the drive of a mine locomotive when braking with a disc brake with a multi-sector disc has been developed. An analytical solution to the non-stationary thermal conductivity problem of finding the temperature field that occurs in the brake disc and friction linings of the disc brake of a mine locomotive when the linings are made in the form of a ring sector was obtained, on the basis of which the dependence of the relative temperature on the friction surface of the brake was found of the disk over time during cyclic braking.

Keywords: *frictional pair, clutch coefficient, disc brake, braking torque, locomotive wheel, rail track.*

Introduction. The adhesion force between locomotive wheels and rails depends both on the state of the rail track and on the conditions of interaction of the wheel-rail friction pair [1]. Much attention is paid to the study of the process of realizing the maximum possible adhesion force. The main parameter characterizing the adhesion force between wheels and rails is the adhesion coefficient. The braking torque created on the wheel by the wheel-block brake depends on the speed of the mine locomotive,

the state of the rail track and the heating of the brake pad, which does not allow the possible adhesion coefficient to be fully realized. Disc brakes used in transport systems do not have this disadvantage [2].

The work [3] presents a method for selecting a constant braking torque applied to the axle of the wheelset. In order to prevent clutch failure and wheel skidding (at the same time, the adhesion force drops sharply and flats form on the wheels), it is recommended for mining electric locomotives to implement 80% of the maximum possible braking torque.

Temperature has the strongest influence on the reliability of the braking device. The temperature stresses arising in the friction pair of a disc brake depend on the intensity of heat generation and cooling during braking, cooling during pauses, braking frequency, the design of the brake unit and the physical properties of the materials of the friction pair. Underestimation of thermal phenomena in the brakes of modern cars can lead to a deviation of their performance characteristics from the calculated ones and even to an accident [2]. With regard to the braking devices of mine locomotives, safety issues come first. Overheating the brake above the maximum permissible temperature can cause an explosion of the methane-air mixture and death. Thus, thermal calculation of the elements of the braking device of any machine operating in a mine is one of the most important tasks in its design.

The monograph [4] considers the problem of heating and cooling the disc brake of mine hoisting machines with a coefficient of mutual overlap between the disc and the friction linings of the pads equal to one.

The purpose of the article is to calculate the maximum temperature and determine the maximum pressure on the working surface for the rational parameters of the main elements of the disk brake of a mine locomotive with a multi-sector brake disk, selected through mathematical modeling.

Main part. Let's consider the choice of rational parameters of a disc brake with a multi-sector brake disc using the example of the E10 mine locomotive. Taking into account the design features of the four-axle mine electric locomotive E10, it is advisable to place a disc brake on the motor shaft of each drive trolley. This will allow two disc brakes to create braking torque on all four axles. By placing disc brakes on the axles of four wheel pairs, their number would double. In addition, the required braking torque on the axle of the wheelset M_T significantly more than the required braking torque on the motor shaft M'_T ($M_T = uM'_T / 2$, where u is a gear ratio of a reducer). Therefore, this would lead to an increase in the geometric dimensions and moment of inertia of the brake discs, or to an increase in their number, i.e. would complicate the design of the braking system and increase its cost.

When calculating frictional devices, the friction coefficient is usually considered as a constant, disregarding its dependence from changing in the course of work of temperature, speed and pressure. Take its smallest possible value for the considered frictional couple for a calculated value of coefficient of friction under existing conditions of work [2].

At determination of the geometrical sizes of a brake disk the internal radius of a working zone is chosen minimum admissible for constructive reasons, and external

radius as it that during creation of the maximum brake moment pressure in a working zone did not exceed admissible value for considered frictional couple [2].

Let's accept quantity of the sectors of a brake disk made in turn of steel 45 HV 415 and the GCI 15-32 HV 200 gray cast iron, equal to eight, pads of the brake shoes made of frictional material 6KH-1 (press material of cold formation) [5] in the form of ring sector with the central corner $\alpha = \pi / 4$. Friction coefficients for the specified couples of materials of a disk and frictional pads are respectively equal to 0.535 and 0.41 [2].

Let's define the maximum necessary moment of braking on an engine shaft $M'_{t\max}$ in the assumption that on the locomotive steel wheels are established. Proceeding from quantity of sectors of a brake disk and a form of frictional pads, we come to a conclusion that dependence of the pulsing braking moment on an engine shaft from the angular coordinate of a shaft of the engine φ_1 can with sufficient degree of accuracy be described by expression

$$\begin{aligned} M'_t &= 2(M_0 - A \sin(n\varphi_2)) / u = M'_0 - A' \sin(n'\varphi_1) = \\ &= M'_0 \left(1 - A^* \sin(n'\varphi_1)\right) = M'_0 \left(1 - \frac{\mu_1 - \mu_2}{\mu_1 + \mu_2} \sin(n'\varphi_1)\right) \quad (\mu_1 > \mu_2), \end{aligned} \quad (1)$$

where M_0, M'_0 are constant components of the moments of braking respectively on an axis of wheel couple and on an engine shaft; n, n' are numbers of the periods of a sinusoid for one turn according to an axis of wheel couple and a shaft of the engine; φ_2 is angular coordinate of an axis of wheel couple; A, A' are amplitudes of fluctuations of variable components of the moments of braking on an axis of wheel couple and on an engine shaft; $A^* = A'/M'_0$; μ_1, μ_2 are friction coefficients for two couples of materials of a disk and frictional pads.

Let's integrate taking into account a formula (1) system of differential equations [5]:

$$\left(\frac{m_s}{4} - m_3 - m_4\right) \ddot{y} = -\left[C_{y3}(y - y_3) + \beta_{y3}(\dot{y} - \dot{y}_3) + C_{y4}(y - y_4) + \beta_{y4}(\dot{y} - \dot{y}_4)\right];$$

$$m_3 \ddot{y}_3 = C_{y3}(y - y_3) + \beta_{y3}(\dot{y} - \dot{y}_3) + F_3(S_3);$$

$$m_4 \ddot{y}_4 = C_{y4}(y - y_4) + \beta_{y4}(\dot{y} - \dot{y}_4) + F_4(S_4);$$

$$I_3 \ddot{\varphi}_3 = -\left[C_{\varphi3}(\varphi_3 - \varphi_2) + \beta_{\varphi3}(\dot{\varphi}_3 - \dot{\varphi}_2) + rF_3(S_3)\right];$$

$$I_4 \ddot{\varphi}_4 = -\left[C_{\varphi4}(\varphi_4 - \varphi_2) + \beta_{\varphi4}(\dot{\varphi}_4 - \dot{\varphi}_2) + rF_4(S_4)\right];$$

$$I_2 \ddot{\varphi}_2 = C_{\varphi3}(\varphi_3 - \varphi_2) + \beta_{\varphi3}(\dot{\varphi}_3 - \dot{\varphi}_2) + C_{\varphi4}(\varphi_4 - \varphi_2) + \beta_{\varphi4}(\dot{\varphi}_4 - \dot{\varphi}_2) - uM'_t/2,$$

where y, y_3, y_4 are linear movements of the locomotive and corresponding wheels; $\dot{y}, \dot{y}_3, \dot{y}_4$ are linear speeds; $\ddot{y}, \ddot{y}_3, \ddot{y}_4$ are linear accelerations; $F_3 = \psi_3(S_3)m_l g/8$, $F_4 = \psi_4(S_4)m_l g/8$ are forces of adhesion of the corresponding wheels;

$$\psi_3 = k_1 \left[th(k_2 S_3) - k_3 S_3 + k_4 S_3^3 \right], \quad \psi_4 = k_1 \left[th(k_2 S_4) - k_3 S_4 + k_4 S_4^3 \right] \quad \text{are}$$

coefficients of coupling of the corresponding wheels (in the mode of braking accept negative values); k_1, k_2, k_3, k_4 are numerical coefficients of the mechanical characteristic of frictional couple; $S_3 = (\dot{\phi}_3 r - \dot{y}_3) / \dot{y}_3$, $S_4 = (\dot{\phi}_4 r - \dot{y}_4) / \dot{y}_4$ are relative slidings of the corresponding wheels; $\ddot{\phi}_2, \ddot{\phi}_3, \ddot{\phi}_4$ are angular accelerations of an output shaft of a reducer and the corresponding wheels; r is radius of a circle of swing of wheels; m_l is mass of the locomotive; g is acceleration of gravity; M'_t is the braking moment on an engine shaft.

When calculating we will use geometrical, weight, elastic, dissipative and rigid characteristics of elements of a mine electric locomotive of E10. Let's accept the mass of structure to the equal mass of the locomotive, i.e. $m_c = m_l = 10^4$ kg. Let's set the initial speed of the engine $v_0 = 1$ m/s. Numerical coefficients of the mechanical characteristic of frictional couple k_1, k_2, k_3, k_4 we will take for a case when rails are sanded [6]. Let's receive that failure of coupling in the course of braking will happen at $M'_0 \geq 766$ N·m. Thus, the maximum value of a constant component of the moment of braking on an engine shaft $M'_{0max} = 766$ N·m. The maximum instantaneous value of the necessary brake moment on an engine shaft

$$M'_{tmax} = M'_{0max} \left(1 + \frac{\mu_1 - \mu_2}{\mu_1 + \mu_2} \right) \quad (\mu_1 > \mu_2).$$

At the chosen materials of sectors of a brake disk and frictional pads $M'_{tmax} = 867$ N·m.

Let's consider a disk brake with one brake disk. For constructive reasons we accept the internal radius of a working zone of a disk $R_1 = 9.3 \cdot 10^{-2}$ m. Let's determine the external radius of a working zone of a disk R_2 . The maximum pressure upon friction surfaces arising during creation of the maximum brake moment M'_{tmax}

$$p_{max} = \frac{M'_{tmax}}{2\mu_{naim} R_e F} = [p],$$

where μ_{naim} is the smallest possible value of coefficient of friction for couple of materials of a disk and frictional pads under existing conditions works;

$R_e = \frac{2(R_2^3 - R_1^3)\alpha}{3(R_2^2 - R_1^2)\sqrt{2(1 - \cos \alpha)}}$ is equivalent radius of friction [2];

$F = \alpha(R_2^2 - R_1^2)/2$ is area of contact of a pad and disk; $[p]$ is the maximum admissible pressure in disk brakes for the considered frictional couple.

After transformations we will receive:

$$R_2 = \sqrt[3]{\frac{6M'_{tmax}\sqrt{2(1 - \cos \alpha)}}{\mu_{naim} \alpha [p] \pi} + R_1^3}.$$

We accept $\mu_{naim} = 0.38$ (taking into account the dependences given in work [7]); $[p] = 8.29 \cdot 10^5 \text{ N/m}^2$ [2]. Then $R_2 = 1.8 \cdot 10^{-1} \text{ m}$.

In work [8] it is shown that the maximum temperature on a surface of friction of a disk reached at the end of braking is stabilized, since the third cycle including braking to a full stop and dispersal. Let's expect temperature surfaces of friction of a brake disk at the end of the third braking to a full stop. We will determine dimensionless temperature on a friction surface in the course of heating by a formula [8]:

$$\theta_{1,2}(\rho, 0, Fo) = \frac{2\pi Bi_{1,2}}{Bi_{1,2}^2 + 1} \sum_{n=1}^{\infty} \frac{V_{01,2}(v_n \rho) (2 + \pi \rho_1 V_{01,2}(\rho_1 v_n))}{v_n (4 - \pi^2 \rho_1^2 V_{01,2}(\rho_1 v_n))} \times \\ \times \int_0^{Fo} Ki(Fo - \tau) \varphi_{1,2}(v_n, \tau) d\tau,$$

where $\theta_{1,2} = (T_{1,2} - T_n)/(T_d - T_n)$ are dimensionless temperatures (hereinafter the index 1 belongs to a disk, the index 2 belongs to frictional pads); $T_{1,2}$ are temperatures; T_n is reference temperature of a disk and pads; T_d is admissible temperature on a friction surface; $\rho = r/R_2$; $\rho_1 = R_1/R_2$; $Fo = a_1 t / R_2^2$ is Fourier's criterion (dimensionless time); $a_{1,2} = \lambda_{1,2} / c_{1,2} \gamma_{1,2}$ are coefficients of heat diffusivity of a disk and frictional pads respectively; $\lambda_{1,2}$ are heat conductivity coefficients; $c_{1,2}$ are specific heat capacities; $\gamma_{1,2}$ are densities; t is time; $Bi_{1,2} = \sigma_{1,2} R_2 / \lambda_{1,2}$ are Biot's criteria; $\sigma_{1,2}$ are heat emission coefficients;

$$V_{01,2}(v_n \rho) = (Bi_{1,2} Y_0(v_n) - v_n Y_1(v_n)) J_0(v_n \rho) + (v_n J_1(v_n) - Bi_{1,2} J_0(v_n)) Y_0(v_n \rho)$$

are kernels of final integral transformation of Hankel on a variable ρ ; Y_0, Y_1, J_0, J_1 are Bessel functions; v_n is the eigenvalues defined from the equation

$$(v_n J_1(v_n \rho_1) + Bi_{1,2} J_0(v_n \rho_1)) (v_n Y_1(v_n) - Bi_{1,2} Y_0(v_n)) - \\ - (v_n J_1(v_n) - Bi_{1,2} J_0(v_n)) (v_n Y_1(v_n \rho_1) + Bi_{1,2} Y_0(v_n \rho_1)) = 0;$$

$Ki = q(t) R_2 / (T_d - T_n) \lambda_1$ is Kirpichev's criterion; $q(t) = \frac{M'_t \omega_n}{t_t F} \int_0^t \left(1 - \frac{\tau}{t_t}\right) d\tau$ is thermal flow; ω_n is the angular speed of a disk in an initial instant; t_t is braking time;

$$\varphi_1 = \alpha_{tp} \kappa e^{-v_n^2 Fo} \left(\frac{1}{\sqrt{\pi Fo}} - (1 - \kappa) Bi_1 e^{\kappa^2 Bi_1^2 Fo} \operatorname{erfc}((1 - \kappa) Bi_1 \sqrt{Fo}) \right); \\ \varphi_2 = \frac{(1 - \alpha_{tp}) \sqrt{a} e^{-a v_n^2 Fo}}{\lambda \sqrt{\pi Fo}};$$

$\alpha_{tp} = \sqrt{\lambda_1 c_1 \gamma_1} / (\sqrt{\lambda_1 c_1 \gamma_1} + \sqrt{\lambda_2 c_2 \gamma_2})$ is the coefficient of distribution of thermal flows showing what part of heat generated at friction is taken away in a brake disk; $\kappa = \alpha / 2\pi$, $erfc x = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-\tau^2} d\tau = 1 - erf x$; $erf x = \frac{2}{\sqrt{\pi}} \int_0^x e^{-\tau^2} d\tau$ is integral of probabilities; $a = a_2 / a_1$; $\lambda = \lambda_2 / \lambda_1$.

As can be seen from the figure, at the final moment of time the dependence of the dimensionless temperature on the friction surface of the brake disc on the dimensionless radius has a maximum at the point $\rho_0 = 0.78$.

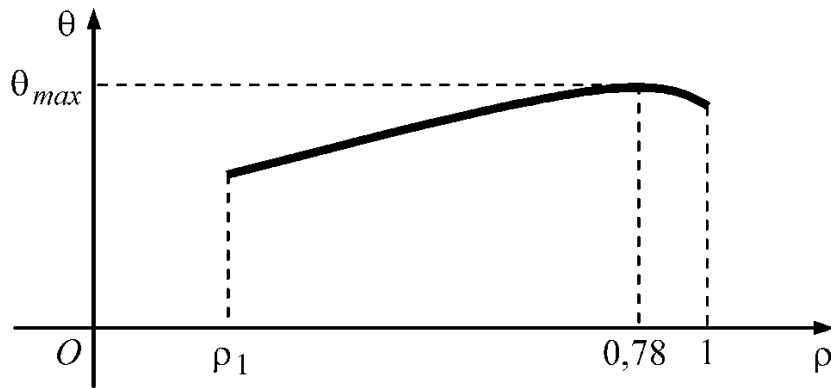


Fig. Dependence of the dimensionless temperature on the surface of the brake disc on the dimensionless radius at the end of braking

During cooling we will determine dimensionless temperature on a surface of friction from a ratio which conclusion is similar provided in the monograph [4]:

$$\theta_{1,2}(\rho, 0, Fo) = \frac{2\pi^2}{Bi_{1,2}^2 + 1} \sum_{n=1}^{\infty} \frac{V_{0,1,2}(v_n \rho)}{4 - \pi^2 \rho_1^2 V_{0,1,2}(v_n \rho_1)} \int_0^{Fo} \eta_{1,2}(v_n, \tau) d\tau + U_{1,2},$$

$$\eta_{1,2} = c_{1,2} \exp \left[-\left(d_{1,2} - Bi_{1,2}^2 \right) \right] \left(\frac{\exp \left(-d_{1,2} Fo \right)}{\sqrt{\pi Fo}} - \sqrt{d_{1,2}} \exp \left(d_{1,2} Fo \right) - 1 \right)$$

where $c_1 = U_1 Bi_1$; $c_2 = \sqrt{a} U_2 Bi_2$; $U_{1,2} = (T_{k1,2} - T_n) / (T_d - T_n)$. At repeated heating instead of T_n it is necessary to substitute the maximum temperature on a surface of friction of a disk at the end of the cooling period. $T_{k1,2}$ is the maximum temperature at the end of the heating period on a friction surface; $d_1 = v_n^2$; $d_2 = a v_n^2$.

Temperature is defined from a ratio

$$T_{1,2} = \theta_{1,2} (T_d - T_n) + T_n.$$

Calculation is feasible in the assumption that the disk is not broken into sectors and is made either of steel 45 HV 415, or of the GCI 15-32 HV 200 gray cast iron at

the following input data: $\omega_n = 201,39$ rad/s (corresponds to the linear speed of the engine $\dot{y} = 5$ m/s); braking time $t_t = 21$ s; dispersal time $t_p = 29$ s; $T_n = 25$ °C; $T_d = 240$ °C. For the disk made of steel 45 HV 415, $a_1 = 1,3 \cdot 10^{-5}$ m²/s; $a_2 = 6,7 \cdot 10^{-8}$ m²/s; $\lambda_1 = 4,5 \cdot 10^1$ Wt/(m·°C); $\lambda_2 = 5,1 \cdot 10^{-1}$ Wt/(m·°C); $c_1 = 461$ J/(kg·°C); $c_2 = 963$ J/(kg·°C); $\sigma_1 = 44$ Wt/(m·°C); $\sigma_2 = 8$ Wt/m·°C. For the disk made of the GCI 15-32 HV 200 gray cast iron, $a_1 = 1,7 \cdot 10^{-5}$ m²/s; $\lambda_1 = 6,3 \cdot 10^1$ Wt/(m·°C); $c_1 = 502$ J/(kg·°C); $\sigma_1 = 44$ Wt/(m·°C). Then the maximum temperature at the end of the third braking on a surface of friction of a brake disk from steel 45 HV 415 $T_1 = 198$ °C, and on a surface of friction of a brake disk from the GCI 15-32 HV 200 gray cast iron is $T_1 = 206$ °C. Thus, taking into account the dependences given in work [7] in specific mine conditions at the end of the third braking to a full stop the maximum temperature on a surface of friction of a multisector disk will not exceed admissible value.

Conclusions. On the basis of mathematical modeling of rational parameters of basic elements of a disk brake of the mine locomotive with a multisector brake disk the maximum temperature and the largest pressure on its working surface are determined. It is established that at the chosen parameters of a disk brake with a multisector disk in specific mine conditions at the end of the third braking to a full stop the maximum temperature on a surface of friction will make no more than 206 °C, i.e. will not exceed admissible value.

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АНОТАЦІЯ

Мета. Для вибраних шляхом математичного моделювання раціональних параметрів основних елементів дискового гальма шахтного локомотива з багатосекторним гальмовим диском обчислити координати максимальної температури та найбільший тиск на робочій поверхні.

Методика. Для знаходження координат максимальної температури і найбільшого тиску на робочій по-верхні дискового гальма з багатосекторним гальмовим диском при вибраних раціональних параметрах було проведено математичне моделювання температури і тиску на поверхні тертя.

Результати. На основі математичного моделювання знайдені максимальна температура та її координати і найбільший тиск на робочій поверхні дискового гальма з багатосекторним гальмовим диском. Показано, що максимальна температура на поверхні тертя основних елементів дискового гальма з вибраними параметрами в специфічних шахтних умовах при найбільш несприятливих умовах роботи не перевищить допустиме значення.

Наукова новизна. Розроблено математичну модель гальмування шахтного локомотива дисковим гальмом, що створює на осі колісної пари пульсуючий гальмовий момент, який залежить від її кутової координати, з урахуванням нелінійної залежності коефіцієнта зчеплення від відносного ковзання, на базі якої встановлені параметри гальмового моменту, що дозволяють поліпшити гальмові характеристики.

Практична значимість. Розроблено науково обґрунтовану інженерну методику вибору раціональних параметрів дискового гальма шахтного локомотива та визначення динамічних і кінематичних характеристик привода шахтного локомотива при гальмуванні дисковим гальмом із багатосекторним диском. Отримано аналітичний розв'язок задачі нестационарної теплопровідності про знаходження температурного поля, що виникає в гальмовому диску та фрикційних накладках дискового гальма шахтного локомотива при виконанні накладок у вигляді кільцевого сектора, на підставі якого знайдена залежність відносної температури на поверхні тертя гальмівного диска від часу при циклічному гальмуванні.

Ключові слова: *фрикційна пара, коефіцієнт зчеплення, дискове гальмо, гальмівний момент, колесо локомотива, рейкова колія.*