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© A. Monia[1]
[1] Ukrainian State University of Science and Technology, Dnipro, Ukraine

MATHEMATICAL MODEL OF VISCOUS INCOMPRESSIBLE FLUID MOTION BETWEEN WHEEL AND RAIL DURING BRAKING AND ACCELERATION

© А.Г. Моня[1]
[1] Український державний університет науки і технологій, Дніпро, Україна

МАТЕМАТИЧНА МОДЕЛЬ РУХУ В'ЯЗКОЇ НЕСТИСЛИВОЇ РІДИНИ МІЖ КОЛЕСОМ І РЕЙКОЮ ПРИ ГАЛЬМУВАННІ І РОЗГОНІ

Purpose. Development, solution and analysis of a mathematical model of the motion of a viscous incompressible fluid in the contact zone of a wheel and a rail to establish the characteristics of rolling a wheel on a rail in the presence of an intermediate medium during braking and acceleration.

The methods. To describe the motion of a viscous incompressible fluid, the Navier-Stokes equations in the polar coordinate system are used. The solution of the system of linear algebraic equations was performed using the Gauss method. To satisfy the velocity and pressure vector projection functions to the boundary conditions, the method of weighted residuals in the form of point-wise collocation was used. Numerical integration was used to determine the lifting force of the intermediate medium and the viscous drag force due to the presence of the intermediate medium as functions of relative sliding.

Findings. The influence of the intermediate medium on the characteristics of wheel-rail adhesion under negative and positive relative slips is substantiated by mathematical modeling of the process of rolling a steel wheel on a rail. It is shown that in order for the ratio of the increase in the relative lifting force of the intermediate medium to the increase in the relative force of viscous resistance compared to the values of these quantities during free rolling to not exceed unity, it is necessary to limit the absolute value of the relative slip to 8.5%.

The originality. For the first time, the influence of an intermediate medium with the properties of a viscous non-compressible fluid on the characteristics of the wheel-rail friction contact under negative and positive relative slips has been substantiated. The relationship between the increase in the relative lift force and the increase in the relative force of viscous resistance on the relative slip was found in comparison with the values of these quantities during free rolling.

Practical implementation. A scientifically based engineering method has been developed for determining the relative lifting force of the intermediate medium, the relative force of viscous resistance caused by the presence of the intermediate medium, and the ratio of the increase in the relative lifting force to the increase in the relative force of viscous resistance compared with the values of these quantities during free rolling as functions of relative slip for given initial data.

Keywords: *frictional couple, coupling coefficient, a locomotive wheel, a railway line, Navier-Stokses equations, a method of the weighed discrepancies.*

Introduction. Steel wheels have relatively stable friction properties and are widely used in rail vehicles and in lifting and transport equipment. The kinematic and dynamic properties of a wheel-rail friction pair are determined by their geometrical parameters, external loads and the presence of an intermediate medium. The rail track in the mines is covered with a significant contaminating fine-dispersed layer, which is a mixture of rock, wear particles of brake pads and wheels in the soil waters. When braking a locomotive, a liquid or multi-dispersed medium located on rails significantly affects the coefficient of adhesion of the wheel to the rail and the rolling resistance force. Currently, the process of interaction of the wheel with the rail in the presence of an intermediate medium has not been studied enough.

In work [1] changes in pressure in the zone of contact between the wheel and the rail were established for various characteristics of the intermediate medium. It is shown that when the load on the wheel of a locomotive moving along a track covered with an intermediate medium changes the carrying capacity of a viscoplastic medium can reduce the coefficient of adhesion to the magnitude of the internal friction of the medium. In this case, the wheel will be in hydroplaning mode. In work [2] on the basis of the equations of the hydrodynamic theory of greasing interaction of a brake shoe wheel-block brakes with a wheel in the presence of the intermediate environment in the form of dispersion of products of wear of lubricants and inorganic pollution in contact zones a block – a wheel and a wheel – a rail is considered.

In work [3], the process of braking of a mine diesel locomotive with a hydrostatic mechanical transmission operating according to the "input differential" scheme is considered. The results of modeling braking due to hydrostatic transmission and the braking system when moving a diesel locomotive in the transport and traction ranges are presented in the form of graphical correlations.

In work [4], it was proven that the pulsating sinusoidal braking torque created on the axis of the wheelset is equal to the sum of the constant component and the oscillation amplitude of the variable component, multiplied by the sine of the product of the number of periods of the sinusoid per one revolution of the wheelset by its angular coordinate, provides higher braking performance than a constant braking torque. It is shown that pulsating braking torque reduces the braking time and braking distance of a mine locomotive.

Main part. Article purpose is development, solution and analysis of a mathematical model of the movement of the wheel along the rail in the presence of an intermediate medium in the contact zone.

The model of the movement of the Newtonian viscous incompressible liquid [5] is applied to establishment of characteristics of swing of a steel wheel on a rail in the presence of the intermediate environment. To the rotating steel wheel on a normal to a rail force which part is perceived by the intermediate environment is applied. In the course of swing of a wheel it is affected by the moment of braking or acceleration M_{ba} (fig. 1).

In fig. 1 the following designations are accepted: R is radius of a circle of driving of a wheel; ω is angular speed of a wheel; $\vec{F}_N$ is normal force; $\vec{N} = \vec{F}_n + \vec{F}_{rl}$; $\vec{F}_n$ is lifting force of the intermediate environment; $\vec{F}_{rl}$ is reaction of a rail; r is current radius; φ is current angular coordinate; h is thickness of a layer of the intermediate environment; $\Delta(z)$ is the gap between a wheel and a rail in the plane $z = const$ (the axis of *Oz* is directed perpendicularly to the drawing plane in such a way that if to look from its end, then positive values of angular movements φ are represented occurring against the course of an hour hand) filled with the intermediate environment; V_{rl} is the speed of a rail of rather geometrical center of a wheel equal on absolute value of speed of the locomotive; 1,2,3,…, 15 are collocation points; θ, θ_1 are the corners defined geometrically

$$\frac{\theta}{2} = \arccos\frac{R - h + \Delta(z)}{R},$$

$$\frac{\theta_1}{2} = \arccos\frac{R - h + \Delta(z)}{OD} = \arccos\frac{R - h + \Delta(z)}{OB_1} =$$

$$= \arccos\frac{\left(R - h + \Delta(z)\right)\cos\frac{\theta}{2}}{OO_1} = \arccos\frac{\left(R - h + \Delta(z)\right)^2}{R\left(R + \Delta(z)\right)}.$$

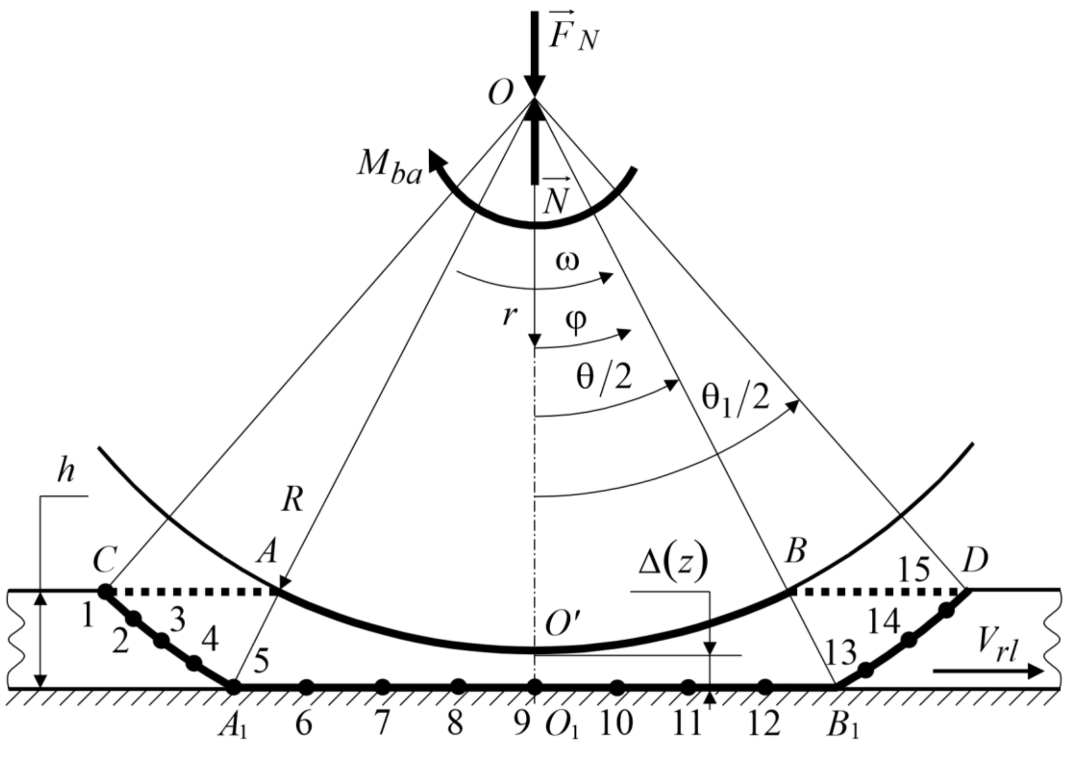

Fig. 1. Design diagram of wheel motion in the presence of an intermediate medium

Neglecting "end effects" and believing that the wheel and a rail have infinite length in the direction of a wheel pivot, we will consider that the movement of the intermediate environment in a gap between a wheel and a rail is flat. Thus, the task is reduced to consideration of the movement of viscous incompressible liquid between the wheel rotating with angular speed ω which geometrical center is not mobile and is a pole O of polar system of coordinates, and the rail moving progressively concerning a pole O in the direction of rotation of a wheel with a speed V_{rl}. Rail speed in the mode of dispersal is less than circumferential speed of a wheel, and in the braking mode – exceeds it. Thus, between working surfaces of a wheel and a rail slipping takes place. Let's use Navier-Stokses equations in polar system of coordinates [5]:

$$\frac{\partial V_r}{\partial t}+V_r\frac{\partial V_r}{\partial r}+\frac{V_\varphi}{r}\frac{\partial V_r}{\partial \varphi}-\frac{V_\varphi^2}{r}=-\frac{1}{\rho}\frac{\partial p}{\partial r}+\nu\left(\nabla^2 V_r-\frac{V_r}{r^2}-\frac{2}{r^2}\frac{\partial V_\varphi}{\partial \varphi}\right);$$

$$\frac{\partial V_\varphi}{\partial t}+V_r\frac{\partial V_\varphi}{\partial r}+\frac{V_\varphi}{r}\frac{\partial V_\varphi}{\partial \varphi}+\frac{V_r V_\varphi}{r}=-\frac{1}{\rho}\frac{1}{r}\frac{\partial p}{\partial \varphi}+\nu\left(\nabla^2 V_\varphi+\frac{2}{r^2}\frac{\partial V_r}{\partial \varphi}-\frac{V_\varphi}{r^2}\right);$$

$$\frac{\partial (rV_r)}{\partial r}+\frac{\partial V_\varphi}{\partial \varphi}=0; \qquad \nabla^2\equiv\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial \varphi^2},$$

where V_r is a velocity vector projection in the direction of the current radius; V_φ is a velocity vector projection in the direction of the current angular coordinate; t is time; ρ is liquid density; p is pressure; ν is kinematic coefficient of viscosity.

The gap between a wheel and a rail is very small in comparison with wheel radius R. We will consider the movement of liquid in a gap slow as inertial members in comparison with the members considering viscous forces and change of pressure can be neglected. Then the linearized Navier-Stokses equations in which there are no inertial members in polar coordinates will take the following form

$$\mu\frac{\partial^2 V_\varphi}{\partial r^2}=\frac{1}{r}\frac{\partial p}{\partial \varphi},\ \mu\frac{\partial^2 V_r}{\partial r^2}=\frac{\partial p}{\partial r},\ \frac{\partial V_r}{\partial r}+\frac{1}{r}\frac{\partial V_\varphi}{\partial \varphi}=0, \tag{1}$$

where $\mu=\nu\rho$ is the dynamic coefficient of viscosity of liquid depending on temperature.

By drawing up these equations the relative trifle of a gap in comparison with wheel radius allowing to consider that is considered

$$V_\varphi>>V_r; \quad \frac{\partial^2 V_\varphi}{\partial r^2}>>\frac{1}{r}\frac{\partial V_\varphi}{\partial r},\ \frac{1}{r^2}\frac{\partial^2 V_\varphi}{\partial \varphi^2},\ \frac{1}{r^2}\frac{\partial V_r}{\partial \varphi},\ \frac{V_\varphi}{r^2};$$

$$\frac{\partial^2 V_r}{\partial r^2}>>\frac{1}{r}\frac{\partial V_r}{\partial r},\ \frac{1}{r^2}\frac{\partial^2 V_r}{\partial \varphi^2},\ \frac{1}{r^2}\frac{\partial V_\varphi}{\partial \varphi},\ \frac{V_r}{r^2}; \quad \frac{\partial V_r}{\partial r}>>\frac{V_r}{r}; \quad \frac{\partial^2 V_r}{\partial r^2}<<\frac{\partial^2 V_\varphi}{\partial r^2}.$$

Follows from the first two equalities of system (1) that

$$r\frac{\partial p}{\partial r}<<\frac{\partial p}{\partial \varphi}.$$

It allows to accept further

$$\frac{\partial p}{\partial r}=0,\ p=p(\varphi).$$

Besides, in system of the equations (1) it is possible to replace out of a derivative sign r on R, and from a variable r $\left(R \le r \le \left(R+\Delta(z)\right)/\cos(\theta/2)\right)$ to pass to the variable $\zeta = r - R$ changing in an interval $0 \le \zeta \le \left(R+\Delta(z)\right)/\cos(\theta/2) - R$. Then $\partial/\partial r = \partial/\partial\zeta$ and Navier-Stokses equations will be written

$$\mu\frac{\partial^2 V_\varphi}{\partial\zeta^2} = \frac{1}{R}\frac{dp}{d\varphi}; \qquad \frac{\partial V_r}{\partial\zeta} = -\frac{1}{R}\frac{\partial V_\varphi}{\partial\varphi}. \tag{2}$$

Normal σ_{rr} and tangent $\tau_{r\varphi}$ stresses according to the generalized law of Newton for incompressible viscous liquid in expanded form in polar system of coordinates according to work [5] are determined by formulas

$$\sigma_{rr} = -p + 2\mu\frac{\partial V_r}{\partial\zeta}; \qquad \tau_{r\varphi} = \mu\left(\frac{1}{R}\frac{\partial V_r}{\partial\varphi} + \frac{\partial V_\varphi}{\partial\zeta} - \frac{V_\varphi}{R}\right). \tag{3}$$

Let's find distribution $V_\varphi = V_\varphi(r,\varphi)$, $V_r = V_r(r,\varphi)$, $p = p(\varphi)$ in the $CABDB_1A_1$ area belonging to the plane $z = const$ (see fig. 1).

At the solution of the specific objectives connected with a flow of firm surfaces viscous liquid boundary conditions have to be used [5]: particles of liquid "stick" to a firm wall, without getting through it, and in a common ground of their speed match speeds of points of a moving firm surface; on removal from a streamline body the speed and pressure, in any point of a flow are set.

Let's write down boundary conditions taking into account that environment speed on border medium – a wheel is equal to wheel speed, on border medium – a rail is equal to rail speed; medium does not get through borders; on removal from a wheel the speed of the environment is equal to rail speed, pressure is equal to zero. Thus,

at $\zeta = 0$, $|\varphi| < \theta/2$ (line $AO'B$)

$$V_\varphi = \omega R = V_{rl}(S+1), \qquad V_r = 0, \tag{4}$$

where $S = \dfrac{\omega R - V_{rl}}{V_{rl}}$ is relative sliding of a wheel on a rail;

at $\zeta = \dfrac{R+\Delta(z)}{\cos\varphi} - R$, $|\varphi| < \theta/2$ (line A_1B_1)

$$V_\varphi = V_{rl}\cos\varphi, \qquad V_r = V_{rl}\sin\varphi; \tag{5}$$

at $\zeta = \dfrac{\left(R+\Delta(z)\right)}{\cos(\theta/2)} - R$, $\theta/2 < |\varphi| < \theta_1/2$ (lines CA_1 and B_1D)

$$V_\varphi = V_{rl}\cos\varphi, \qquad V_r = V_{rl}\sin\varphi, \qquad p = 0. \tag{6}$$

The approximation of the decision satisfying to Navier-Stokses equations (2) and to boundary conditions (4) it is identical, we will choose in a look

$$\left.\begin{aligned}
&V_\varphi = \omega R + \left(a\zeta - \zeta^2\right)\left(\sum_{i=1}^{n} a_i \cos\frac{2i\varphi\pi}{\theta_1} + \sum_{i=1}^{k} b_i \sin\frac{2i\varphi\pi}{\theta_1}\right);\\
&V_r = \frac{2\pi}{R\theta_1}\left(\frac{\zeta^3}{3} - \frac{a\zeta^2}{2}\right)\left(-\sum_{i=1}^{n} a_i i\sin\frac{2i\varphi\pi}{\theta_1} + \sum_{i=1}^{k} b_i i\cos\frac{2i\varphi\pi}{\theta_1}\right) + f(\varphi);\\
&p = -\frac{\mu R\theta_1}{\pi}\left(\sum_{i=1}^{n}\frac{a_i}{i}\sin\frac{2i\varphi\pi}{\theta_1} - \sum_{i=1}^{k}\frac{b_i}{i}\cos\frac{2i\varphi\pi}{\theta_1}\right),
\end{aligned}\right\} \tag{7}$$

where

$$a = \Delta(z)\left(1 - \frac{2}{\pi}\operatorname{arctq}S\right);$$

$$f(\varphi) = \begin{cases} V_{rl}\sin\left(\theta_1(\varphi + \theta/2)/(\theta_1 - \theta)\right), & \text{at} \quad -\theta_1/2 \le \varphi < -\theta/2;\\ 0, & \text{at} \quad |\varphi| \le \theta/2;\\ V_{rl}\sin\left(\theta_1(\varphi - \theta/2)/(\theta_1 - \theta)\right), & \text{at} \quad \theta/2 < \varphi \le \theta_1/2. \end{cases}$$

We will also define unknown coefficients a_i and b_i so that the chosen approximation of the decision met boundary conditions (5), (6). For satisfaction of functions V_φ and V_r to boundary conditions (5) and (6), and also function p to a boundary condition (6) we will use method of the weighed discrepancies in the form of a pointwise collocation [6]. We will choose points of a collocation on the CA_1B_1D line asymmetrically rather direct $\varphi = 0$.

From system of the equations (7) we have

$$\sum_{i=1}^{n} K_{ji} a_i + \sum_{i=1}^{k} M_{ji} b_i = L_j, \tag{8}$$

where

$$K_{ji} = \cos\frac{2i\varphi_j\pi}{\theta_1},\ M_{ji} = \sin\frac{2i\varphi_j\pi}{\theta_1},\ L_j = \frac{V_p\left(\cos\varphi_j - S - 1\right)}{a\zeta_j - \zeta_j^2}$$

for the first equation of system (7);

$$K_{ji} = -i\sin\frac{2i\varphi_j\pi}{\theta_1},\ M_{ji} = i\cos\frac{2i\varphi_j\pi}{\theta_1},\ L_j = \frac{R\theta_1\left(V_p\sin\varphi_j - f(\varphi_j)\right)}{2\pi\left(\frac{\zeta_j^3}{3} - \frac{a\zeta_j^2}{2}\right)}$$

for the second equation of system (7);

$$K_{ji} = \frac{1}{i}\sin\frac{2i\varphi_j\pi}{\theta_1},\ M_{ji} = -\frac{1}{i}\cos\frac{2i\varphi_j\pi}{\theta_1},\ L_j = 0$$

for the third equation of system (7);

$$\zeta_j = \frac{R+\Delta(z)}{\cos\varphi_j} - R$$

on the line A_1B_1;

$$\zeta_j = \frac{R+\Delta(z)}{\cos(\theta/2)} - R$$

on lines CA_1 и B_1D; $j = 1, 2, 3, ..., m$ (m – total quantity of the equations of system (8)).

The total number of unknown a_i and b_i has to be equal in system of the linear algebraic equations (8) to number of the equations. Thus, the number of members of ranks in decomposition (7) depends on quantity of points of a collocation. For carrying out numerical calculations we will take 15 points of a collocation. Points on an entrance to the $CABDB_1A_1$ area we will arrange more densely, than at the exit (see fig. 1). Then the system (8) will consist of thirty-eight equations and it is possible to accept n = 19, k = 19. Considering $R = f(z) = const$, $\Delta(z) = const$, we will determine the carrying power of the intermediate environment and force of the viscous resistance caused by existence of the intermediate environment as functions of relative sliding on formulas

$$F_n = b\int\limits_{AB} \sigma_{rr}\cos\varphi\, dl = \frac{b\mu R^2\theta_1^2}{\pi}\sum_{i=1}^{n}\frac{b_i}{i}\left(\frac{1}{2i\pi-\theta_1}\sin\frac{(2i\pi-\theta_1)\theta}{2\theta_1} + \right.$$

$$\left. + \frac{1}{2i\pi+\theta_1}\sin\frac{(2i\pi+\theta_1)\theta}{2\theta_1}\right); \tag{9}$$

$$F_c = b\int\limits_{AB} \tau_{r\varphi}\cos\varphi\, dl = bR\mu\left(a\theta_1\sum_{i=1}^{n} a_i\left(\frac{1}{2i\pi-\theta_1}\sin\frac{(2i\pi-\theta_1)\theta}{2\theta_1} + \right.\right.,$$

$$\left.\left. + \frac{1}{2i\pi+\theta_1}\sin\frac{(2i\pi+\theta_1)\theta}{2\theta_1}\right) + 2\omega\sin\frac{\theta}{2}\right), \tag{10}$$

where b is width of a zone of contact of a wheel and intermediate environment; $dl = \sqrt{r^2 + r'^2}\,d\varphi = Rd\varphi$ is curve arch differential.

The decision of system of the linear algebraic equations (8) was executed by the Gauss method for the sizes of relative sliding changing in the range from minus units (the skid mode) to two (slipping drafts in the mode with the district speed of a wheel three times exceeding rail speed concerning a wheel). Further taking into account formulas (9) and (10) the relative carrying power of the intermediate environment $F_n^* = F_n/F_N$, the relative force of viscous resistance caused by existence of the intermediate environment $F_c^* = F_c/F_N$ and the relation of increase in relative carrying power to increase in relative force of viscous resistance in comparison with values of these sizes at free swing were found $F_n^* = F_n / F_N$, $F_c^* = F_c / F_N$

$$F_\Delta^* = \frac{F_n^*(S) - F_n^*(0)}{F_c^*(S) - F_c^*(0)}$$

as functions of relative sliding at the following input data: R = 0.27 m; V_{rl} = 5 m/s; $h = 5 \cdot 10^{-3}$ m; $\Delta(z) = 10^{-3}$ m; $b = 10^{-2}$ m; $F_N = 1.25 \cdot 10^4$ N; μ = 5.214 N · s/m^2. Calculations were carried out by means of a standard package of the application programs "Mathematica" for 15 points of a collocation (see fig. 1). Increase in number of points of a collocation (more than 15) significantly will not influence the decision. It speaks about good convergence of ranks (7).

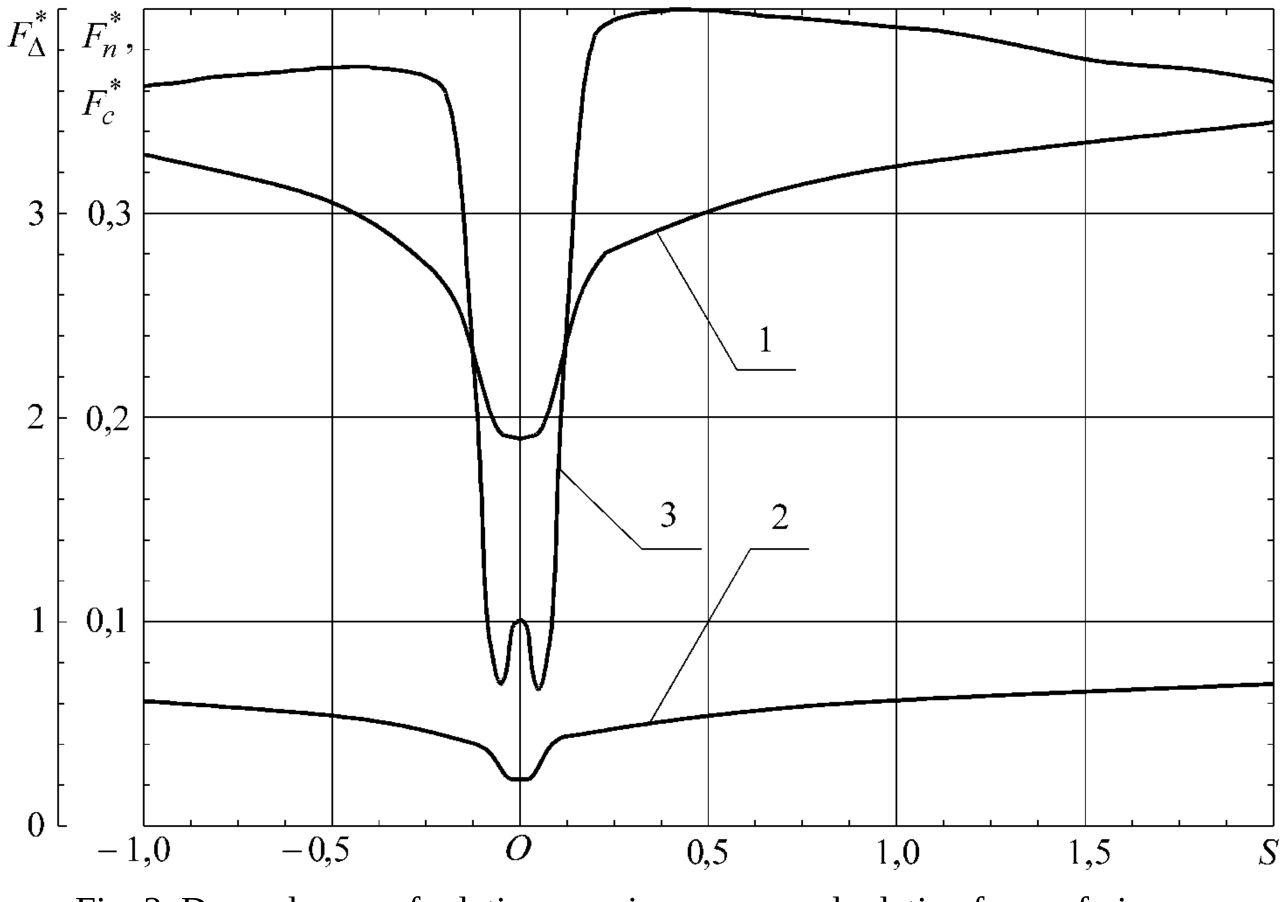

Fig. 2. Dependences of relative carrying power and relative force of viscous resistance on relative sliding: 1 is relative carrying power F_n^* ; 2 is relative force of viscous resistance F_c^* ; 3 is relation of increase in relative carrying power to increase in relative force of viscous resistance F_Δ^*

It is visible (see fig. 2) that dependences of relative carrying power and relative force of viscous resistance increase with increase $|S|$. Moreover, on an interval $0 < |S| < 0.05$ increase of function $F_c^* = F_c^*(S)$ reaches bigger size, than function increase $F_n^* = F_n^*(S)$. At the relative sliding, equal $\pm 0{,}05$, function $F_\Delta^* = F_\Delta^*(S)$ has minima ($F_{\Delta\min}^* = 0.62$ and $F_{\Delta\min}^* = 0.64$ respectively). On an interval $0.05 < |S| < 0.2$ the function graph $F_n^*(S)$ has significantly sharper rise, than a function graph $F_c^*(S)$. On this interval of value of function $F_\Delta^*(S)$ increase in the braking mode by 5.6 times (with 0.64 to 3.60), passing through unit at $S = -0.085$ and in the dispersal mode by 6.2 times (with 0.62 to 3.87), passing through unit at $S = 0.085$. It promotes reduction of an absolute value of coefficient of coupling ψ. Further, at $0.2 < |S| < 1$ value of function $F_\Delta^*(S)$ changes slightly and makes about 3,65 in the mode of braking and 3.9 in the dispersal mode. At $1 < S < 2$ value of function $F_\Delta^*(S)$ it is approximately equal to 3.85. Function $F_\Delta^*(S)$ at $0 < |S| < 0.1$ in the mode of dispersal accepts smaller values, than in the braking mode, and at $0.1 < |S| < 1$ on the contrary larger values.

Conclusions. On the basis of the carried-out calculations and the analysis it is established that in the presence of the intermediate environment in the modes of dispersal and braking relative sliding differently influences coefficient of coupling of wheels with rails

For stabilization of coefficient of coupling ψ during dispersal and braking in the presence between a wheel and a rail of the intermediate environment it is necessary to limit absolute value of relative sliding of 8.5%.

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АНОТАЦІЯ

Мета. Розробка, розв'язання та аналіз математичної моделі руху в'язкої нестисливої рідини в зоні контакту колеса та рейки для встановлення характеристик кочення колеса по рейці за наявності проміжного середовища в процесі гальмування та розгону.

Методика. Для опису руху в'язкої нестисливої рідини використані рівняння Нав'є-Стокса в полярній системі координат. Розв'язання системи лінійних алгебраїчних рівнянь виконано методом Гауса. Для задоволення функцій проекцій вектора швидкості та тиску граничним умовам використаний метод зважених нев'язок у вигляді поточкової колокації. При визначенні підйомної сили проміжного середовища та сили в'язкого опору, обумовленого наявністю проміжного середовища, як функцій відносного ковзання застосовано чисельне інтегрування.

Результати. Обґрунтовано вплив проміжного середовища на характеристики зчеплення колеса з рейкою при від'ємному і додатному відносних ковзаннях шляхом математичного моделювання процесу кочення сталевого колеса по рейці. Показано, що для того, щоб відношення збільшення відносної підйомної сили проміжного середовища до збільшення відносної сили в'язкого опору в порівнянні зі значеннями цих величин при вільному коченні не перевищувало одиниці, необхідно обмежувати абсолютне значення відносного ковзання величиною 8,5%.

Наукова новизна. Вперше обґрунтовано вплив проміжного середовища, що має властивості в'язкої нестисливої рідини, на характеристики фрикційного контакту колесо-рейка при від'ємному і додатному відносних ковзаннях. Знайдено залежність відношення збільшення відносної підйомної сили до збільшення відносної сили в'язкого опору відносного ковзання порівняно зі значеннями цих величин при вільному коченні.

Практична значимість. Розроблено науково обґрунтовану інженерну методику визначення відносної підйомної сили проміжного середовища, відносної сили в'язкого опору, обумовленого наявністю проміжного середовища, і відношення збільшення відносної підйомної сили до збільшення відносної сили в'язкого опору в порівнянні зі значеннями цих величин при вільному коченні як функцій відносного ковзання при заданих вихідних даних.

Ключові слова: *фрикційна пара, коефіцієнт зчеплення, колесо локомотива, рейкова колія, рівняння Навье-Стокса, метод зважених нев'язок.*