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© O. Pashchenko[1], Ye. Koroviaka[1], V. Khomenko[1], O. Davydenko[1]
[1] Dnipro University of Technology, Dnipro, Ukraine

MATHEMATICAL MODEL OF DRILLING MUD FILTRATION IN A POROUS MEDIUM TAKING INTO ACCOUNT DYNAMIC CHANGES IN PARAMETERS

© О.А. Пащенко[1], Є.А. Коровяка[1], В.Л. Хоменко[1], О.М. Давиденко[1]
[1] Національний технічний університет «Дніпровська політехніка», Дніпро, Україна

МАТЕМАТИЧНА МОДЕЛЬ ФІЛЬТРАЦІЇ БУРОВОГО РОЗЧИНУ В ПОРИСТОМУ СЕРЕДОВИЩІ З УРАХУВАННЯМ ДИНАМІЧНИХ ЗМІН ПАРАМЕТРІВ

Purpose. The aim of the study is to develop a mathematical model of drilling mud filtration in a cellular medium, taking into account dynamic changes in its parameters and interaction with rocks.

Methods. Analytical and numerical modeling methods were used to study filtration processes, including a system of differential equations, a modified Darcy's law, the continuity equation, and viscosity dependence on pressure. The numerical solution was implemented using the finite difference method, which effectively describes the dynamic filtration processes. The effects of time, temperature, and solid phase concentration on changes in the permeability of the porous medium were considered. Model verification was performed by comparing the obtained results with experimental data.

Results. The proposed model accounts for nonlinear effects associated with changes in viscosity, permeability, and pressure gradient under real drilling conditions. An analysis was conducted on the impact of temperature, mechanical, and chemical clogging on the filtration process. The formation of a filter cake on the wellbore walls and its impact on drilling mud circulation losses were investigated. A numerical analysis of scenarios involving changes in drilling mud characteristics and their influence on drilling stability was carried out.

Scientific novelty. A mathematical model of drilling mud filtration has been developed, incorporating nonlinear and non-stationary effects not previously considered in classical approaches. The model introduces permeability dependence on time and solid phase concentration, allowing for more accurate predictions of filter cake formation. For the first time, the interrelation between filtration rate and changes in the porous medium structure due to clogging has been examined.

Practical significance. The proposed model can be used to optimize the parameters of the drilling fluid, which will reduce the risks of complications during drilling, such as circulation losses and leachate penetration into the productive layer. The obtained results can be applied in the design of drilling flushing systems and improvement of filtration control methods in the oil and gas industry.

Keywords: *filtration, drilling fluid, permeability, viscosity, clogging, mathematical modeling.*

Introduction. Filtration of drilling fluid into a porous medium is an important process in drilling oil and gas wells [1–3]. It has a significant impact on the stability of the wellbore walls, flow hydrodynamics, formation and structure of the filter cake, and prevention of circulation losses. Incorrect filtration control can lead to various complications, such as fluid penetration into the productive formation, reduced permeability of the bottomhole zone, and deterioration of the conditions for subsequent well operation [4, 5].

Traditional filtration models are based on Darcy's law and a number of empirical relationships, but they do not always adequately describe the real process, since under drilling conditions the key parameters of the solution and porous medium change in time and space. In particular, the viscosity of the drilling fluid depends on pressure and temperature, the permeability of the porous medium changes due to colmatation (blockage of pores by particles), and the filter cake is formed dynamically depending on the characteristics of the solid phase of the solution. These factors require the development of a mathematical model that takes into account nonlinear and non-stationary effects [6].

An important problem is to take into account the interaction of drilling fluid with rock under conditions of changing pressure gradient, mechanical and chemical colmatation, as well as dynamic changes in the structure of the filter cake [7–9]. The complexity of modeling lies in the need to describe the relationship of hydrodynamic, rheological and colloid-chemical processes occurring in the "drilling fluid – porous medium" system [10].

Analysis of existing drilling mud filtration models. Filtration of drilling fluid into a porous medium is a complex physical and chemical process that depends on many factors, such as solution viscosity, rock permeability, pressure gradient, temperature, and filter cake properties. Various mathematical models are used to describe this process, which can be divided into classical (based on fundamental physical laws) and modern (taking into account nonlinear effects, dynamic changes in parameters, and numerical solution methods) [11–13].

The basis of most filtration models is Darcy's law, established in 1856 [14], which describes the linear relationship between filtration rate and pressure gradient:

$$\upsilon = -\frac{k}{\mu}\nabla P\,, \tag{1}$$

where: υ – the filtration rate, m/s; k – the rock permeability coefficient, m²; μ – the dynamic viscosity of the drilling fluid, Pa s; P – the pressure, Pa.

This law is valid for stationary, incompressible, and Newtonian fluids at low pressure gradients. However, in drilling environments, these assumptions are often not met, which limits the applicability of Darcy's model.

Another important classical model is the Cauchy equation [15], which describes the law of conservation of mass for a fluid in a porous medium:

$$\frac{\partial(\phi p)}{\partial t} + \nabla\cdot(p\upsilon) = 0\,, \tag{2}$$

where: ϕ – the porosity of the rock; ρ – the density of the drilling fluid.

This equation considers the change in liquid density and can be used to describe the filtration of compressible solutions.

It is also necessary to consider the Reynolds number, which determines the flow regime of liquid in a porous medium:

$$\mathrm{Re} = \frac{p\upsilon d}{\mu}, \tag{3}$$

where: *d* – the characteristic pore size.

When the flow is assumed to be laminar *Re < 10*, Darcy's law is applicable. However, at high pressure gradients, nonlinear effects may occur, requiring modification of the model.

Classical filtration models do not consider nonlinear processes that occur in real drilling conditions, such as changes in drilling mud viscosity, pore space clogging, and filter cake formation. Extended nonlinear filtration models are used to more accurately describe the process:

A number of studies introduce a correction factor into the Darcy equation that takes into account the effect of the dependence of permeability on the pressure gradient:

$$\upsilon = -\frac{k}{\mu}\nabla P + \beta\left|\nabla P\right|, \tag{4}$$

where: β – an empirical coefficient depending on the characteristics of the environment.

To consider the rheological properties of drilling fluid, models with changing viscosity are used, which relate viscosity to shear stress:

$$\mu = \mu_0\left(1 + \alpha\left|\nabla P\right|\right), \tag{5}$$

where: α – the coefficient of viscosity sensitivity to the pressure gradient.

Colmatation (clogging of pores with the solid phase of the drilling fluid) leads to a change in permeability over time. This process can be described by the equation:

$$\frac{\partial k}{\partial t} = -\gamma \cdot C_f \cdot \upsilon, \tag{6}$$

where: C_f – the concentration of solid particles in the solution, γ – the colmatation coefficient.

Modern approaches include the numerical solution of a system of nonlinear differential equations describing filtration considering dynamic changes in parameters. For this purpose, finite difference, finite element, or molecular dynamics methods are used.

Filtration of drilling fluid in real conditions is accompanied by changes in its rheological parameters, permeability of the porous medium and pressure gradient over time. This leads to the following key effects:

Depending on temperature and pressure, the viscosity of the drilling fluid may change, which affects the filtration rate. For example, with an increase in temperature, the viscosity of polymer solutions decreases, which accelerates filtration.

During colmatation, solid particles settle in the pores of the rock, reducing its permeability. This leads to a decrease in the filtration rate and an increase in the thickness of the filter cake [16].

In real conditions, the pressure in the well changes during the drilling process, which can lead to pulsations of the filtration flow and local changes in the characteristics of the drilling fluid [17].

All these factors require consideration in the mathematical model of filtration to ensure an adequate description of real processes and optimize drilling parameters to minimize complications.

Purpose of the article. The aim of this study is to develop a mathematical model of drilling fluid filtration into a porous medium, considering dynamic changes in parameters such as solution viscosity, rock permeability, pressure and filtration rate.

To achieve this goal, the article solves the following tasks:

Analysis of existing mathematical models of filtration and identification of their shortcomings under conditions of changing parameters.

Formulation of a system of differential equations describing the filtration process taking into account the dynamics of changes in the parameters of the drilling fluid and the porous medium.

Development of a numerical method for solving a system of equations and conducting computational experiments.

Verification of the model by comparison with experimental data and existing models.

Analysis of the obtained results and formulation of recommendations for the application of the model to optimize drilling processes.

Thus, the mathematical model will improve the accuracy of forecasting filtration processes and will help in choosing the optimal parameters of drilling fluid for specific drilling conditions.

Materials and research methods. Filtration of drilling mud occurs in a porous medium represented by a rock, which is characterized by a system of microscopic channels (pores) distributed in a solid matrix. The geometry of the pore space depends on the type of rock: intergranular porosity is typical for sandstones, a fractured-cavernous structure is typical for carbonate rocks, and micropores and capillaries are typical for clay deposits. The study considers a porous medium with a uniform distribution of pores, which allows the use of continuum filtration models.

To simplify the modeling, it is assumed that the porous medium is isotropic, i.e. its properties do not depend on the direction. In addition, it is assumed that there are no macroscopic cracks and faults in the studied area that can significantly change the structure of the fluid flow. Filtration is considered at a scale at which individual pores are not explicitly modeled, and their influence is averaged through macroscopic parameters such as permeability coefficient and porosity.

The model is based on a number of assumptions that simplify the mathematical description of the process. It is assumed that the drilling fluid is an incompressible liquid, which corresponds to real conditions at relatively low filtration rates and no significant pressure changes. Temperature effects are considered weakly expressed in the short-term time interval, so the temperature is assumed to be constant in the basic model, although its consideration can be included if necessary.

The effect of colmatation is considered through the dynamic change in the permeability coefficient of the porous medium. The colmatation process is associated with the deposition of solid particles of the drilling mud on the walls of the pores, which leads to a decrease in the effective diameter of the pore space and a decrease in the filtration capacity of the medium. In the model, colmatation is considered through the empirical dependence of the change in permeability on the concentration of solid particles in the solution and the filtration rate.

The main parameters that determine the filtration process are the viscosity of the drilling fluid, the pressure in the porous medium, the permeability coefficient, and the time-spatial variables. The viscosity of the drilling fluid depends on its composition and filtration conditions, and it can change over time due to interaction with the rock. The pressure in the porous medium changes under the action of the filtration flow and external loads, which requires considering the pressure gradient in modeling.

The permeability coefficient is a function of time and coordinates, as it changes under the influence of colmatation. The time dependence of the model allows for the dynamics of the filtration process to be considered, including transient processes when changing the system parameters. Spatial variables include the coordinates of the study area, which determine the direction and distribution of the drilling fluid flow in the porous medium.

Results. Filtration of drilling mud in a porous medium considering dynamic changes in parameters is described by a system of equations including the law of fluid motion, the equation of conservation of mass, and empirical dependencies linking viscosity, pressure, and permeability. The main complexity of the model is in considering nonlinear effects associated with colmatation, viscosity changes, and pressure gradient.

The basis of the mathematical model is the equation of motion of the filtration flow. In its classical form, Darcy's law describes the motion of an incompressible fluid in a porous medium as a linear relationship between the filtration rate and the pressure gradient. However, under conditions of changing permeability and viscosity, its modification is required:

$$v = -\frac{k(r,t)}{\mu(p)}\nabla p, \tag{7}$$

where: v – the filtration flow rate; $k(r,t)$ – the permeability coefficient, depending on the coordinate r and time t; $\mu(p)$ – the dynamic viscosity of the drilling fluid, depending on the pressure p; ∇p – the pressure gradient.

The continuity equation describes the conservation of mass of a fluid in a porous medium (fig. 1):

$$\frac{\partial(\phi p)}{\partial t} + \nabla \cdot (p v) = 0, \tag{8}$$

where: ϕ – the porosity of the medium; ρ – the density of the drilling mud; v – the filtration flow rate.

The relationship between viscosity and pressure through empirical relationships (fig. 1): The viscosity of the drilling fluid can change depending on the pressure. This relationship can be described by an empirical formula, for example:

$$\mu(p) = \mu_0 e^{ap}, \tag{9}$$

where: μ_0 – the initial viscosity at zero pressure; α – the coefficient that determines the dependence of viscosity on pressure.

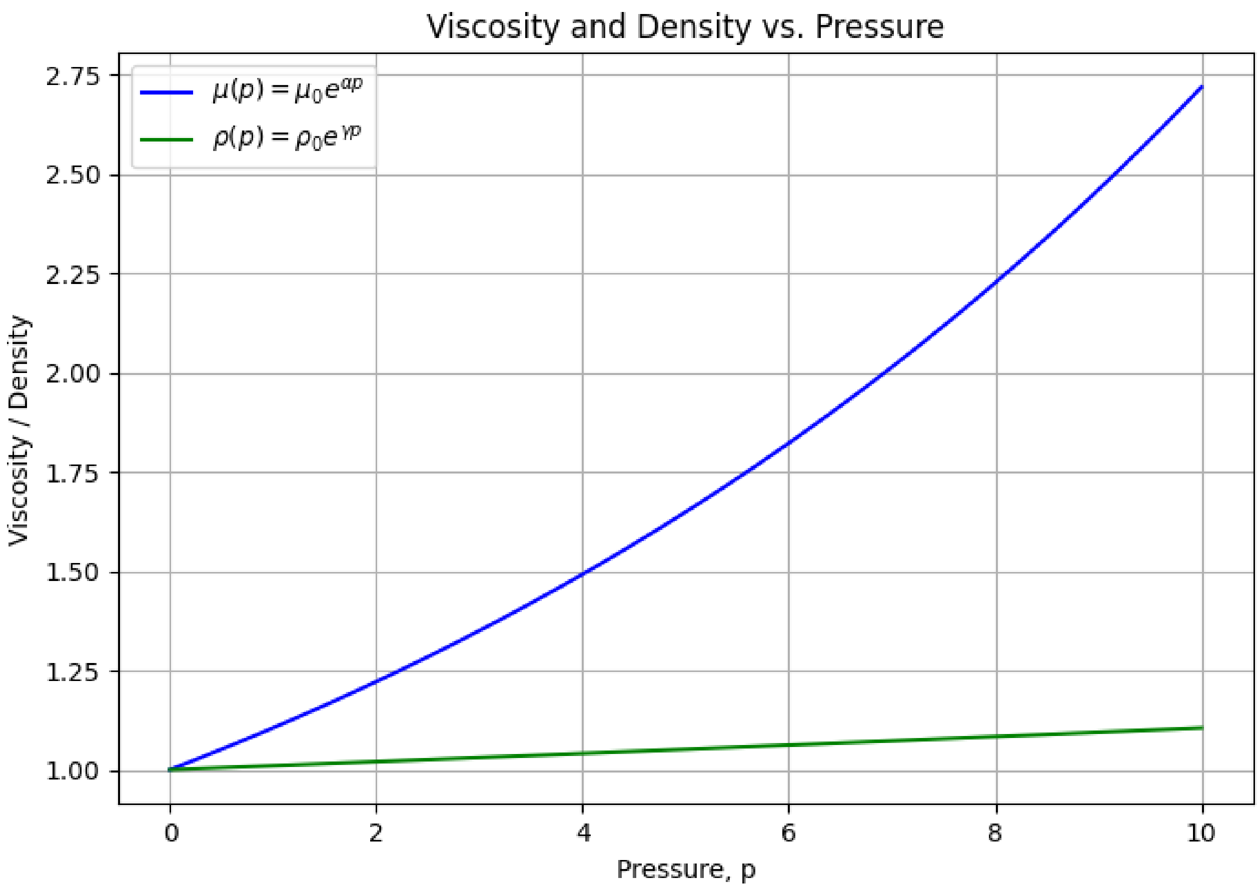

Fig. 1. Mathematical model of filtration flow in a porous medium
(Viscosity and density as functions of pressure)

Correction of the permeability coefficient taking into account colmatation (fig. 2): Colmatation (blockage of pores) leads to a decrease in permeability. This can be taken into account using the relationship:

$$k(r,t) = k_0 e^{-\beta t}, \tag{10}$$

where: k_0 – the initial permeability; β – the coefficient determining the rate of colmatation.

Combining the equations of motion, continuity and dependence of parameters, we obtain a system of differential equations:

Modified Darcy equation:

$$v = \frac{k_0 e^{-\beta t}}{\mu_0 e^{ap}} \nabla p. \tag{11}$$

Continuity equation:

$$\frac{\partial(\phi p)}{\partial t} + \nabla \cdot \left(p \left(-\frac{k_0 e^{-\beta t}}{\mu_0 e^{ap}} \nabla p \right) \right) = 0. \tag{12}$$

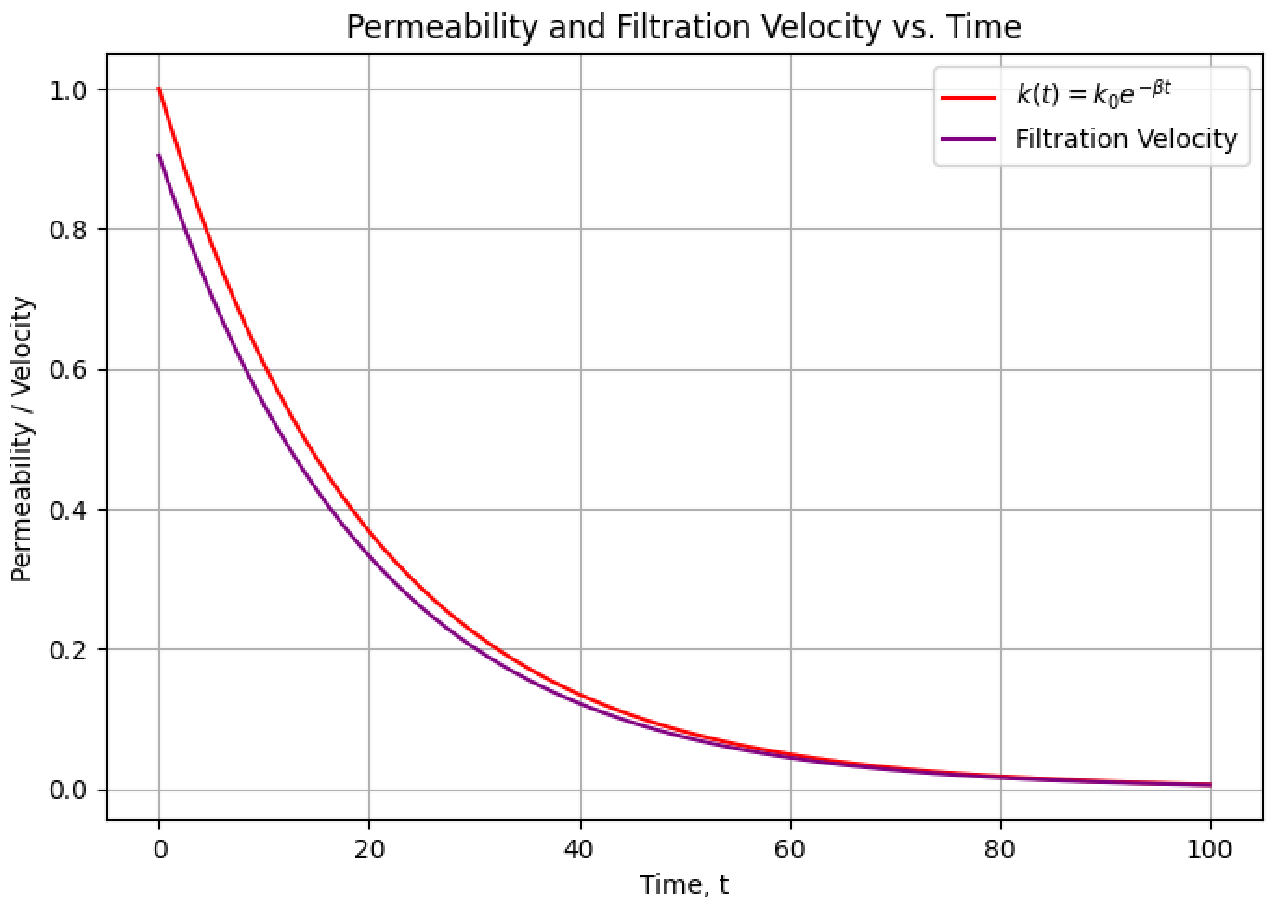

Fig. 2. Mathematical model of filtration flow in a porous medium
(Permeability and filtration velocity as functions of time)

Equation of state: The density of drilling fluid can depend on pressure:

$$p = \rho_0 e^{yp}, \tag{13}$$

where: ρ_0 – the initial density; γ – the compressibility coefficient.

Substituting the equation of state into the continuity equation, we obtain:

$$\frac{\partial\left(\phi\rho_0 e^{yp}\right)}{\partial t} + \nabla\cdot\left(\rho_0 e^{yp}\left(-\frac{k_0 e^{-\beta t}}{\mu_0 e^{ap}}\nabla p\right)\right) = 0. \tag{14}$$

Simplifying, we obtain the final system of equations:

$$\phi\gamma\frac{\partial p}{\partial t} - \nabla\cdot\left(\frac{k_0 e^{-\beta t}}{\mu_0 e^{ap}}\nabla p\right) = 0. \tag{15}$$

To solve the system of equations numerically, we set the initial and boundary conditions. The numerical solution of the model requires the use of finite difference or finite element methods, which will allow us to take into account the dynamics of the filtration process in a porous medium (fig. 3, 4).

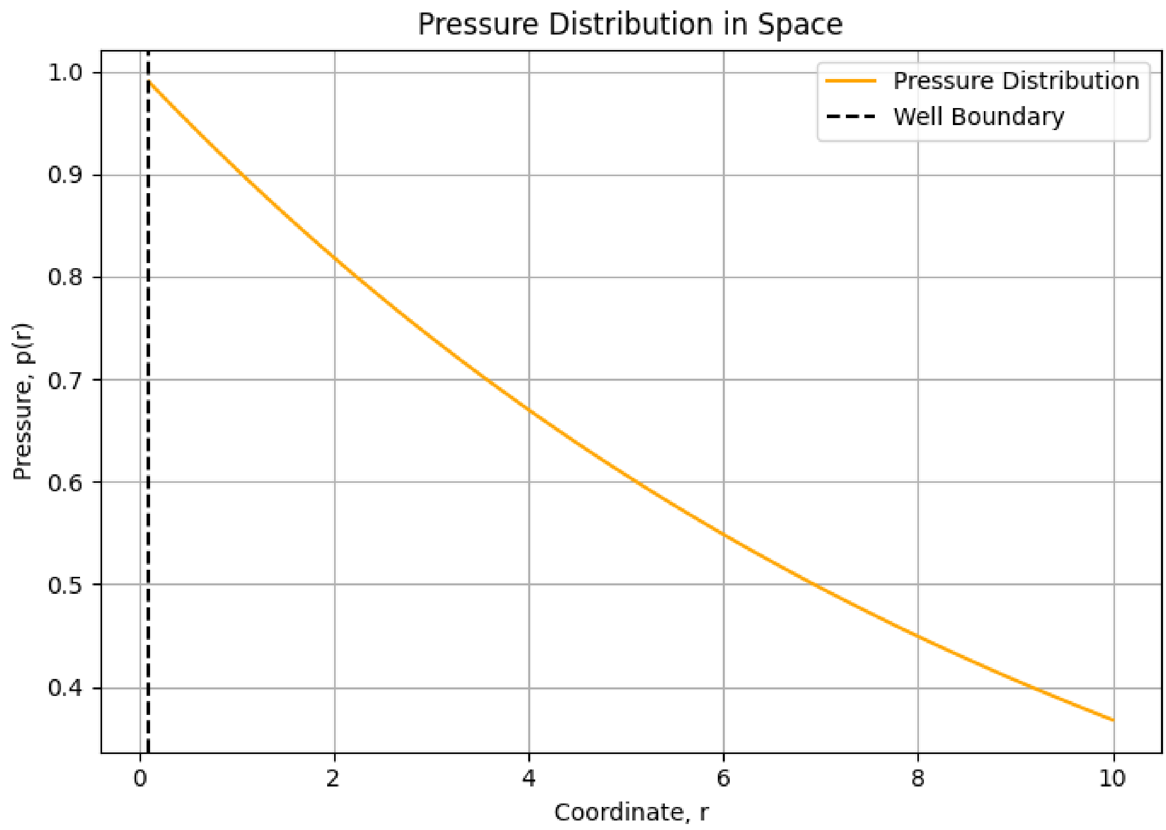

Fig. 3. Mathematical model of filtration flow in a porous medium
(Pressure distribution as a function of radial coordinate)

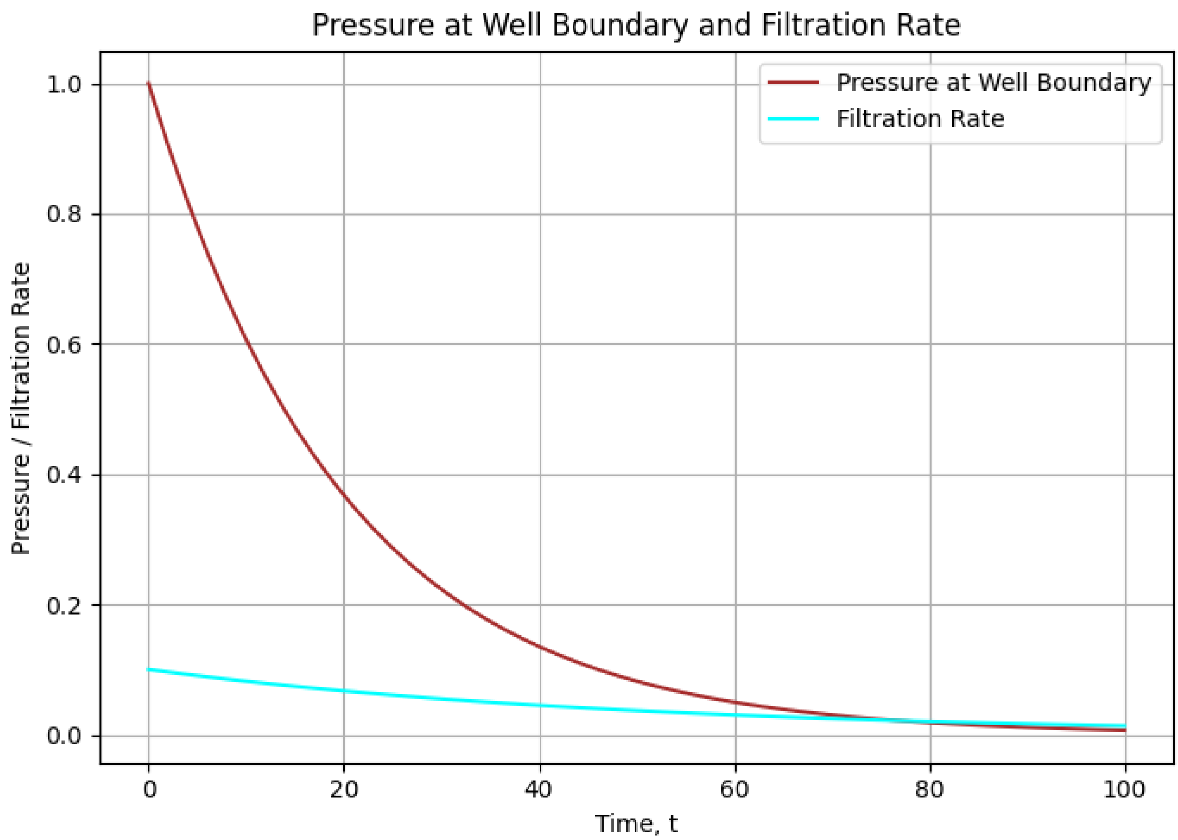

Fig. 4. Mathematical model of filtration flow in a porous medium
(Pressure at the well boundary and filtration rate as functions of time)

Dirichlet: fixed pressure at the boundaries of the study area.

Neumann: a given pressure gradient or fluid flow.

Mixed conditions: a combination of boundary conditions at different parts of the boundary.

To numerically solve a system of equations, it is necessary to specify boundary and initial conditions.

Initial conditions:

Pressure at the initial moment of time:

$$p(r, t = 0) = p_0(r). \tag{16}$$

where: $p_0(r)$ – the initial pressure distribution.

Permeability at the initial moment of time:

$$k(r, t = 0) = k_0. \tag{17}$$

Boundary conditions:

At the well boundary (for example, at): $r = r_w$

$$p(r_w, t) = r_w(t), \tag{18}$$

where: $p_w(t)$ – the pressure on the well wall, which can change over time.

At the outer boundary (for example, at $r = r_e$):

$$\frac{\partial p}{\partial r}\Big|_{r=r_e} = 0, \tag{19}$$

which corresponds to the absence of flow across the outer boundary.

Additional conditions:

Accounting for the filtration of the solution across the boundary:

$$v \cdot n = q(t), \tag{20}$$

where: $q(t)$ – the filtration rate through the boundary, n – the normal to the boundary.

Based on the constructed graphs (see. fig. 1–4), conclusions can be drawn about the behavior of the filtration flow in the porous medium. The viscosity of the drilling mud and its density increase exponentially with increasing pressure, which is associated with the compressibility of the liquid. As the pressure increases, the liquid molecules are compressed, which leads to an increase in both viscosity and density. The permeability of the medium and the filtration rate decrease exponentially with time due to colmatation, that is, clogging of pores, which hinders the flow of fluid through the porous medium. The pressure in the porous medium decreases with increasing distance from the well, reaching a maximum value near the well and gradually decreasing with distance from it. The pressure on the well wall and the filtration rate through the boundary also decrease with time, which reflects the effect of well operation on its productivity. These results confirm that the mathematical model adequately describes the key filtration processes, such as the dependence of viscosity and density on pressure, a decrease in permeability over time, the distribution of pressure in space and the dynamics of pressure and filtration rate at the well boundary.

Discussion. The developed mathematical model of drilling mud filtration in a porous medium takes into account dynamic changes in parameters such as permeability, viscosity and pressure. To verify the adequacy of the model, comparisons were made with experimental data, as well as with the results of other well-known models, such as the classical Darcy model and the Forchheimer model [18].

The experimental data included measurements of pressure distribution, permeability and filtration rate in a porous medium under various conditions. The comparison showed that the developed model agrees well with the experimental data, especially in the area of colmatation and viscosity change depending on pressure. However, some discrepancies were found in areas with high pressure gradients, indicating the need to refine the model for such cases [19].

Metrics such as mean square error (MSE) and coefficient of determination R^2 were used to quantify the accuracy of the model. The results showed that the model provides high accuracy in most cases but requires additional tuning to improve predictive ability in extreme conditions.

To assess the robustness of the model and its applicability under different conditions, a sensitivity analysis was performed. Key model parameters such as initial permeability k_0, colmatation coefficient β, pressure-permeability coefficient γ, characteristic transition length λ, and drilling fluid viscosity μ_0 were varied over a wide range.

The Monte Carlo method was used to evaluate the impact of random parameter variations on the model results. The analysis showed that the initial permeability and the colmatation coefficient are the most sensitive parameters, having the greatest impact on the pressure and permeability distribution. The drilling fluid viscosity and the pressure-permeability coefficient have a smaller impact, but are also important for the accuracy of the model.

One of the key objectives of the work was to predict the filtration dependencies that may arise when changing drilling fluid characteristics, such as viscosity, density, and solids content. For this purpose, various scenarios for changing drilling fluid characteristics were considered.

For example, increasing the viscosity of the drilling fluid leads to a decrease in the filtration rate, which can cause pressure to accumulate near the borehole wall. Increasing the solids content increases the colmatation, which leads to a decrease in permeability and an increase in pressure. Modeling these scenarios allows us to determine the critical values of the parameters at which anomalies occur, such as a sharp decrease in permeability or an increase in pressure.

Conclusions. The study developed a mathematical model of drilling mud filtration in a porous medium, taking into account dynamic changes in parameters such as permeability, viscosity and pressure. The main results include the development of a system of equations, including a modified Darcy equation, a continuity equation and empirical dependencies for viscosity and permeability. The numerical solution of the system of equations was implemented using the finite difference method, and boundary and initial conditions were specified for the correct solution of the problem. Verification of the model showed its high accuracy in most cases, although areas requiring additional adjustment were identified.

The developed model has important practical significance for the oil and gas industry. It can be used to optimize the drilling process, control the quality of drilling mud and design wells. The model allows predicting the distribution of pressure and permeability, which helps reduce the risk of anomalies and improve drilling efficiency. In addition, the model can be useful for training specialists, helping them better understand filtration processes and their impact on the drilling process.

Several lines of research are proposed to further improve the model and expand its capabilities. First, it is necessary to take into account nonlinear effects, such as the nonlinear dependence of viscosity and permeability on pressure and temperature. Second, it is important to develop methods for modeling filtration in complex geological conditions, such as fractured and layered rocks. Third, integrating the model with other models, such as heat transfer and rock mechanics models, will allow for a more comprehensive analysis of drilling processes. Additional experimental studies will help to refine the model parameters and test its adequacy in various conditions. Finally, developing specialized software for automating calculations and visualizing modeling results will make the model more accessible for practical use.

The developed mathematical model of drilling mud filtration in a porous medium has demonstrated high accuracy and applicability for describing real processes. Its practical application includes drilling optimization, drilling mud quality control, and well design. Further research is aimed at improving the model, taking into account nonlinear effects, and integrating it with other models for a more comprehensive analysis of drilling processes. These efforts will improve the efficiency and safety of drilling processes in the oil and gas industry.

References

1. Koroviaka, Ye.A., Mekshun, M.R., Ihnatov, A.O., Ratov, B.T., Tkachenko, Ya.S., & Stavychnyi, Ye.M. (2023). Determining technological properties of drilling muds. *Naukovyi Visnyk Natsionalnoho Hirnychoho Universytetu, 2,* 25–32. https://doi.org/10.33271/NVNGU/2023-2/025
2. Khomenko, V.L., Ratov, B.T., Pashchenko, O.A., Davydenko, O.M., Borash, B.R. (2023). Justification of drilling parameters of a typical well in the conditions of the Samskoye field. *IOP Conference Series: Earth and Environmental Science, 1254* (2023), 012052. https://doi.org/10.1088/1755-1315/1254/1/012052
3. Pavlychenko, A.V., Ihnatov, A.O., Koroviaka, Y.A., Ratov, B.T., & Zakenov, S.T. (2022). Problematics of the issues concerning development of energy-saving and environmentally efficient technologies of well construction. *IOP Conference Series: Earth and Environmental Science, 1049* (2022), 012031. https://doi.org/10.1088/1755-1315/1049/1/012031
4. García-Chan, N., Alvarez-Vázquez, L. J., Martínez, A., & Vázquez-Méndez, M. E. (2025). A non-conservative macroscopic traffic flow model in a two-dimensional urban-porous city. *Mathematics and Computers in Simulation, 233*, 60–74. https://doi.org/10.1016/j.matcom.2025.01.016
5. Chudyk, I.I., Femiak, Ya.M., Orynchak, M.I., Sudakov, A.K., & Riznychuk, A.I. (2021). New methods for preventing crumbling and collapse of the borehole walls. *Naukovyi Visnyk Natsionalnoho Hirnychoho Universytetu, 4*, 17–22. https://doi.org/10.33271/nvngu/2021-4/017
6. Davydenko, A.N., Kamyshatsky, A.F., & Sudakov, A.K. (2015). Innovative technology for preparing washing liquid in the course of drilling. *Science and Innovation, 11*(5), 5–13. https://doi.org/10.15407/scine11.05.005

7. Pashchenko, O., Khomenko, V., Ishkov, V., Koroviaka, Y., Kirin, R., & Shypunov, S. (2024). Protection of drilling equipment against vibrations during drilling. *IOP Conference Series: Earth and Environmental Science, 1348* (2024) 012004. https://doi.org/10.1088/1755-1315/1348/1/012004
8. Pashchenko, O., Ratov, B., Khomenko, V., Gusmanova, A., & Omirzakova, E. (2024). Methodology for optimizing drill bit performance. *International Multidisciplinary Scientific GeoConference Surveying Geology and Mining Ecology Management, SGEM, 24*(1.1), 623–631. https://doi.org/10.5593/sgem2024/1.1/s06.78
9. Mamatova, H., Eshkuvatov, Z., & Ismail, S. (2025). Homotopy perturbation method for semi-bounded solution of the system of Cauchy-type singular integral equations of the first kind. *Journal of Advanced Research in Applied Sciences and Engineering Technology, 51*(2), 124–137. https://doi.org/10.37934/araset.51.2.124137
10. Al-Obaidi, A. K., Majdi, H. Sh., Jweeg, M. J., Hadi, F. A., Jasim, D. J., & Ellafi, A. (2024). Artificial intelligence for real-time prediction of rheological drilling mud properties. *Iraqi Geological Journal, 57*(1), 147–161. https://doi.org/10.46717/igj.57.1E.10ms-2024-5-21
11. Ali, I., Ahmad, M., & Lashari, N. (2024). Optimizing filtration properties of water-based drilling mud systems using dually modified starch. *Journal of Cleaner Production, 454*, 142022. https://doi.org/10.1016/j.jclepro.2024.142022
12. Wiedemann, D., & Peter, M. A. (2025). A Darcy law with memory by homogenisation for evolving microstructure. *Journal of Mathematical Analysis and Applications, 546*(2), 129222. https://doi.org/10.1016/j.jmaa.2025.129222
13. Ramadan, A. M., Osman, A., Mehanna, A., Shehata, A. I., & Shehadeh, M. (2024). Simulation of filter-cake formations on vertical and inclined wells under elevated temperature and pressure. *SPE Journal, 29*(5), 2212–2224. https://doi.org/10.2118/219446-PA
14. Duan, Y., Dong, X., Yang, H., Fan, Y., Ma, X., & Lin, W. (2024). Study of solid-liquid two-phase flow model of drilling fluids for analyzing mud cake formation. *Geoenergy Science and Engineering, 236*, 212761. https://doi.org/10.1016/j.geoen.2024.212761
15. Huang, H., Li, J., Gao, R., Zhang, G., Yang, H., Chen, W., Luo, M., & Li, W. (2023). Investigation of the mechanisms and sensitivity of wellbore breathing effects during drilling in deepwater shallow formations. *Ocean Engineering, 269*, 113405. https://doi.org/10.1016/j.oceaneng.2022.113405
16. Pashchenko, O.A., Khomenko, V.L., Ratov, B.T., Koroviaka, Ye.A., & Rastsvietaiev, V.O. (2024). Comprehensive approach to calculating operational parameters in hydraulic fracturing. *ICSF-2024. IOP Conf. Series: Earth and Environmental Science 1415*(2024), 012080. https://doi.org/10.1088/1755-1315/1415/1/012080
17. Davydenko, O., Ratov, B., & Ighnatov, A. (2016). Determination of basic calculation and experimental parameters of device for bore hole cleaning. *Mining of Mineral Deposits, 10*(3), 52–58. https://doi.org/10.15407/mining10.03.052
18. Oseh, J. O., Norddin, M. N. A. M., Ismail, I., Duru, U. I., Gbadamosi, A. O., Agi, A., Ngouangna, E. N., Blkoor, S. O., Yahya, M. N., & Risal, A. R. (2023). Rheological and filtration control performance of water-based drilling muds at different temperatures and salt contaminants using surfactant-assisted novel nanohydroxyapatite. *Geoenergy Science and Engineering, 228*, 211994. https://doi.org/10.1016/j.geoen.2023.211994
19. Wang, D., Qiu, Z., Miao, H., Geng, T., Zhong, H., Zhao, X., & Fan, L. (2022). Study on property control of high-density drilling fluids based on modified Alferd model. *Drilling Fluid and Completion Fluid, 39*(6), 692–699. https://doi.org/10.12358/j.issn.1001-5620.2022.06.005

АНОТАЦІЯ

Мета. Метою дослідження є розробка математичної моделі фільтрації бурового розчину в пористому середовищі з урахуванням динамічних змін його параметрів та взаємодії з гірськими породами.

Методика. Використано аналітичні та чисельні методи моделювання процесів фільтрації, що включають систему диференційних рівнянь, модифікований закон Дарсі, рівняння неперервності та залежності в'язкості від тиску. Чисельне розв'язання здійснювалося методом скінченних різниць, що дозволяє ефективно описати динамічні процеси фільтрації. Враховано вплив часу, температури та концентрації твердої фази на зміну проникності пористого середовища. Виконано верифікацію моделі шляхом порівняння отриманих результатів із експериментальними даними.

Результати. Запропонована модель дозволяє враховувати нелінійні ефекти, пов'язані зі змінами в'язкості, проникності та градієнта тиску в реальних умовах буріння. Виконано аналіз впливу температури, механічної та хімічної кольматації на процес фільтрації. Досліджено формування фільтраційної кірки на стінках свердловини та її вплив на втрати циркуляції бурового розчину. Проведено чисельний аналіз сценаріїв зміни характеристик бурового розчину та їхнього впливу на стабільність буріння.

Наукова новизна. Розроблено математичну модель фільтрації бурового розчину, яка враховує нелінійні та нестаціонарні ефекти, що раніше не розглядалися у класичних підходах. Введено залежність проникності від часу та концентрації твердої фази, що дозволяє точніше прогнозувати процес утворення фільтраційної кірки. Вперше досліджено взаємозв'язок між швидкістю фільтрації та зміною структури пористого середовища під впливом кольматації.

Практична значущість. Запропонована модель може бути використана для оптимізації параметрів бурового розчину, що дозволить знизити ризики ускладнень під час буріння, таких як втрати циркуляції та проникнення фільтрату в продуктивний пласт. Отримані результати можуть бути застосовані при проектуванні бурових промивальних систем та вдосконаленні методик контролю фільтрації у нафтогазовій промисловості.

Ключові слова: *фільтрація, буровий розчин, проникність, в'язкість, кольматація, математичне моделювання.*