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COMPARATIVE ANALYSIS OF COLUMN BASE CALCULATION UNDER BENDING MOMENT IN IDEA STATICA CONNECTION AND THE SEMI-ANALYTICAL FINITE ELEMENT METHOD (SAFEM)

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ПОРІВНЯЛЬНИЙ АНАЛІЗ РОЗРАХУНКУ БАЗИ КОЛОНИ ПРИ ДІЇ ЗГИНАЛЬНОГО МОМЕНТУ В IDEA STATICA CONNECTION ТА НМСЕ

Purpose. To perform a comparative analysis of the results of calculating a column base under bending moment obtained using four approaches: the Semi-Analytical Finite Element Method (SAFEM), the software package Idea StatiCa Connection, the software package LIRA-SAPR, and analytical calculation, as well as to establish the physical causes of discrepancies between the methods.

Methodology. The study was carried out based on calculations of a column base with 30K1 section on a plate 490×490×20 mm under bending moment M = 10 kN·m. SAFEM uses volumetric elements with contact interaction through the zonal method, Idea StatiCa uses shell elements with Winkler model, LIRA-SAPR uses volumetric elements. Analytical calculation was performed for a cantilever plate with triangular foundation pressure diagram.

Results. Satisfactory convergence of forces in anchor bolts between all methods was established — the discrepancy does not exceed 14%. Significant dependence of stresses in the base plate on the adopted calculation scheme was revealed: Idea StatiCa and LIRA-SAPR give close results — 35.2 and 37.6 MPa with 6% discrepancy, while SAFEM gives 15.9 MPa due to the use of volumetric finite elements for a thin plate (t/L = 0.04). Analytical calculation explains the discrepancies obtained by different cantilever overhang of the plate.

Scientific novelty. For the first time, a comprehensive comparative analysis of four methods for calculating column bases was performed. It was established that bolt forces demonstrate high convergence between methods, while plate stresses depend on the type of finite elements — the difference reaches 2.4 times. It was shown that shell elements are physically more justified for thin plates, which is confirmed by analytical calculation.

Practical significance. The obtained results allow a design engineer to make an informed choice of calculation approach when designing steel column bases, taking into account the features of each method. The established patterns contribute to improving the calculation methods for column-to-foundation connections and increasing the accuracy of assessing their stress-strain state.

Keywords: *column base, stress-strain state, Semi-Analytical Finite Element Method (SAFEM), Idea StatiCa Connection, LIRA-SAPR, bending moment, cantilever plate.*

Introduction. Steel column bases are critical nodes of load-bearing structures in industrial and civil buildings, subjected to a complex stress-strain state arising from axial forces, bending moments, and shear forces. Accurate assessment of stresses in the base plate and forces in the anchor bolts is essential for ensuring the reliability and safety of the structure as a whole [1–9].

Modern engineering practice involves the use of specialized software for the calculation of steel structural connections [4, 8]. Software products such as Idea StatiCa Connection and LIRA-SAPR have gained wide adoption, implementing different approaches to modeling the stress-strain state of connections. Alongside these, semi-analytical methods are being actively developed – in particular, the Semi-Analytical Finite Element Method (SAFEM), which has demonstrated effectiveness in solving contact problems [1, 3, 7, 10, 11].

At the same time, the question of convergence of results obtained by different methods remains insufficiently studied [12, 13]. A design engineer using different software packages may obtain substantially different results without understanding the reasons for such discrepancies. This necessitates a comparative analysis aimed at identifying the physical causes of the differences between methods.

The objective of this work is a comparative analysis of the results of steel column base calculation under bending moment, obtained using SAFEM, Idea StatiCa Connection [6], LIRA-SAPR [9], and analytical calculation, and the identification of the causes of discrepancies between them [10].

1. Problem Statement.

The subject of the study is the stress-strain state of the connection node between a steel column and its foundation — the column base — under the action of a bending moment in the plane of maximum cross-sectional stiffness. The structural model comprises a steel I-section column 30K1, a base plate, and four anchor bolts connecting the plate to a reinforced concrete foundation (fig. 1).

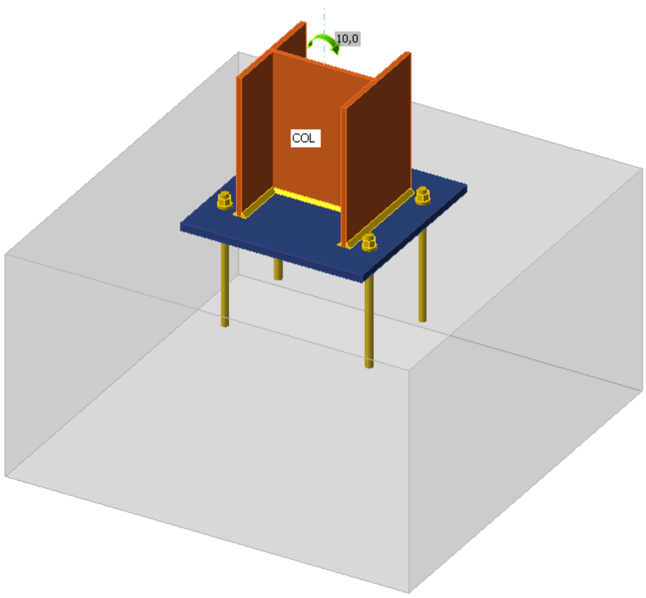

Fig. 1. Calculation scheme

Geometric parameters of the problem: base plate dimensions 490×490 mm, plate thickness $t = 20$ mm; anchor bolts are placed symmetrically about the plate center at distances of $e_1 = 125$ mm and $e_0 = 195$ mm from the axes of symmetry. Cross-sectional properties of the 30K1 column: $h = 294$ mm, $b = 294$mm, $t_w = 7$mm, $t_f = 12$ mm. The arrangement of the bolts and the I-section in plan view is shown in fig. 2.

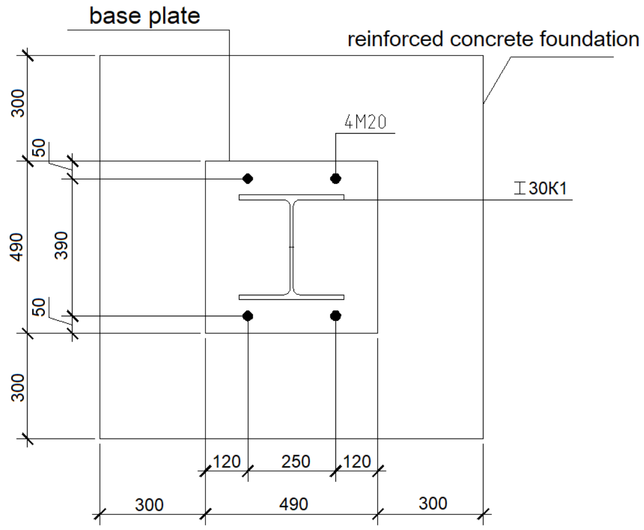

Fig. 2. Anchor bolt and column layout plan

The external load is a bending moment $M = 10kN*m$ with no axial force ($N = 0$). The contact interaction between the base plate and the foundation is modeled as unilateral — only compressive stresses are transferred, and separation of the plate from the foundation surface is not permitted.

The problem formulation differs depending on the method applied. In Idea StatiCa Connection, the 30K1 column is modeled in full as an assembly of shell finite elements, which ensures direct transfer of the moment through the web and flanges of the cross-section to the base plate (see Fig. 1). In LIRA-SAPR and SAFEM, explicit modeling of the column cross-section was not performed — the transfer of the bending moment from the column to the plate is implemented as a pair of concentrated forces applied at the anchor bolt nodes. The force in the pair is determined from the moment equilibrium condition:

$$F = M / a = 10 \times 10^6 / 390 = 25.6kN,$$

where $a = 2 \times e_2 = 390$ mm is the distance between the bolts along the moment action axis. In LIRA-SAPR, the anchor bolts are modeled as bar finite elements; in SAFEM, the contact interaction between the plate and the foundation is implemented using the zonal method [11]. In the analytical calculation, the base plate is treated as a cantilever plate, fixed along the perimeter of the column cross-section [12], with a triangular distribution of the reactive foundation pressure.

$$\sigma_{\max} = M / W = 0.51\text{МПа}.$$

2. Calculation Method

Semi-Analytical Finite Element Method (SAFEM). In the adopted formulation, the displacement field along the structural axis is approximated by an expansion in Lagrange polynomials, while standard FEM discretization is used in the cross-section. The base plate and foundation are modeled using solid finite elements. The contact interaction between the plate and the foundation is accounted for by the zonal method, which iteratively identifies contact and separation zones by varying the elastic modulus of the contact elements [3]. The SAFEM structural model is shown in fig. 3.

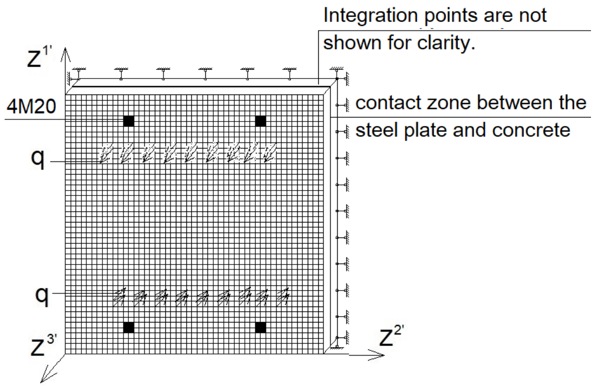

Fig. 3. SAFEM calculation scheme

Idea StatiCa Connection. The software implements the finite element method based on the Component-Based Finite Element Method (CBFEM) [5, 6]. All connection elements — the column, base plate, and welds — are modeled using shell finite elements. The contact interaction between the base plate and the foundation is described by a Winkler elastic foundation model with unilateral springs acting in compression only [2]. The structural model is shown in Fig. 1.

LIRA-SAPR. The 20 mm thick base plate is modeled using solid finite elements — 20 layers of elements through the thickness (1 mm per layer). The foundation is modeled using planar finite elements CE-44 resting on an elastic foundation [2, 9]. The anchor bolts are modeled as bar finite elements. The bending moment from the column is transferred to the plate through a pair of oppositely directed distributed loads applied as strips of 1 mm width along the flange lines of the 30K1 I-section. The contact interaction between the plate and the foundation is implemented through corresponding boundary conditions [9]. The structural model is shown in fig. 4.

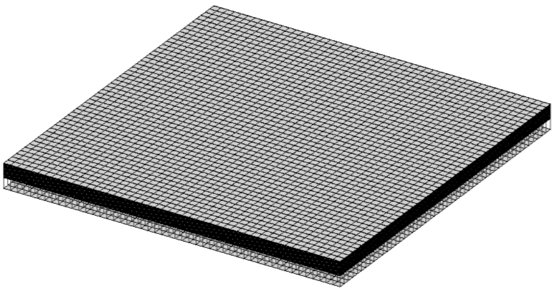

Fig. 4. LIRA-SAPR calculation scheme

Analytical Calculation. The base plate is treated as a cantilever plate, fixed along the perimeter of the column cross-section and loaded from below by the reactive foundation pressure. With $N = 0$ and $M = 10 kN \times m$. The stresses in the plate are determined from the cantilever bending formula with cantilever projection $c = 490/2 - 294/2 = 245 - 147 = 98 mm$ (from the column flange to the plate edge), giving $\sigma = 37.7MPa$. For a strip of 1 mm width: $q = 0.51MPa \times 1mm = 0.51N/mm$, the stress is

$$\sigma = 6 \times M / t^2 = 6 \times 2447 / 20^2 = 36.7MPa$$

The bolt force $F = M / h = 10 \times 10^6 / 294 = 34.01kN$ (for the pair); per single bolt $F_1 = 34.01 / 2 = 17.0kN$

3. Calculation Results and Their Analysis

The calculation of the column base in the Idea StatiCa Connection software package was performed based on the normative documents EN 1993-1-8: Eurocode 3 [5]. The calculation model is shown in fig. 1. The results of calculating forces in anchor bolts are presented in fig. 4. The base plate stress values are shown in fig. 5 — $\sigma = 35.2MPa$

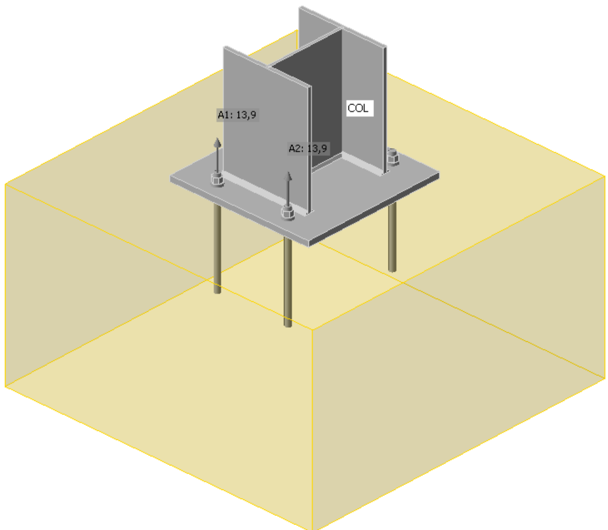

Fig. 5. Results of calculating forces in anchor bolts (Idea StatiCa)

The computational model in the LIRA-SAPR software is shown in fig. 4. The anchor bolt force results are presented in fig. 6. the base plate displacements are shown in fig. 7.

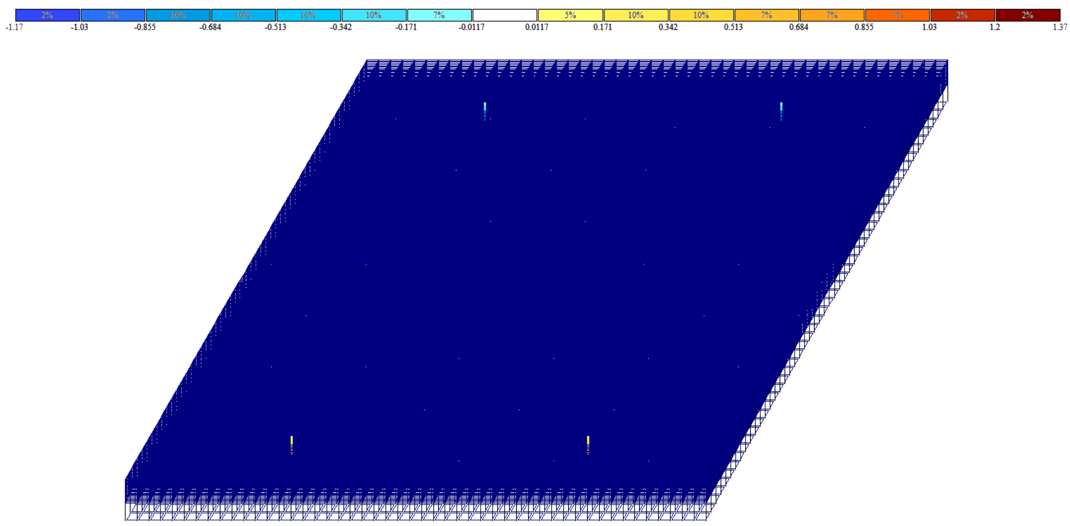

Fig. 6. Results of calculating forces in bolts (LIRA-SAPR)

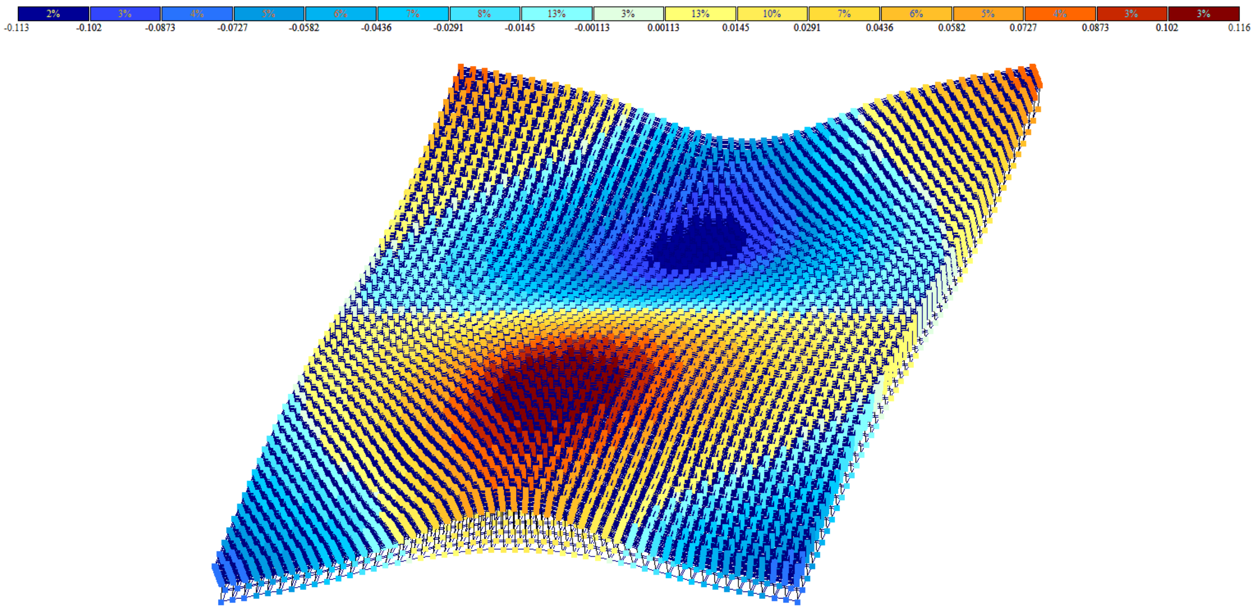

Fig. 7. Base plate displacements (LIRA-SAPR)

The base plate stress results (FEM) are shown in fig. 8.

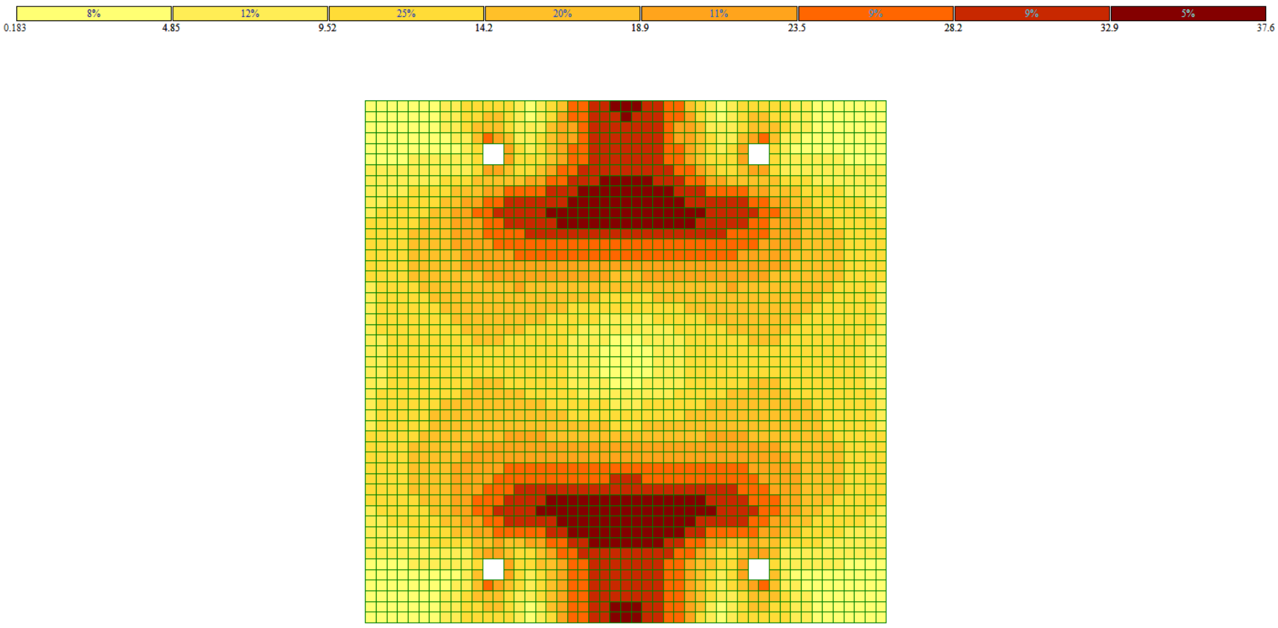

Fig. 8. Stresses in the plate (LIRA-SAPR)

Based on the LIRA-SAPR calculation results, a local stress concentration is observed in the base plate directly at the anchor bolts, where the maximum value reaches 52 MPa. However, these stresses are a consequence of the modeling approach in which the bolts are represented as bar finite elements with a point connection to the plate, which does not correspond to the actual contact interaction between the bolt, nut, and washer. In the comparative analysis, the stresses in the cantilever working zone of the plate between the column and the bolts are considered, where the maximum value is 37.6 MPa. Similarly, in

Idea StatiCa Connection, local peak stresses near the connection elements are not considered when evaluating the overall stress-strain state of the plate. The combined results of the calculation by all four methods are presented in table.

Table

Summary of overall calculation results

	FEM	SAFEM	Idea StatiCa	Analytical calculation
Bolt force, kN	13,69	15,8	13.9	17.0
Base plate stress, MPa	37.6	15.9	35.2	36.7
Base plate displacement, mm	0.116	0.123	-	-

According to the calculation results, the maximum force in a single anchor bolt obtained from Idea StatiCa is 13.9 kN, from LIRA-SAPR – 13.69 kN, with a discrepancy of 1.5% between them. The SAFEM value is 15.8 kN, which differs from the Idea StatiCa result by 14%. The analytical bolt force is determined from the moment equilibrium condition: $F = M/h = 10 \cdot 10^6/294 = 34.01$ kN for the bolt pair, i.e. 17.0 kN per single bolt. The obtained results indicate satisfactory agreement among all four approaches in determining bolt forces.

Regarding the base plate stresses, the results of the methods differ substantially. According to the Idea StatiCa calculation, the maximum stress in the base plate is 35.2 MPa; in LIRA-SAPR – 37.6 MPa, with a discrepancy of 6% between them. The SAFEM value is 15.9 MPa, which is nearly half the results of the two software packages. The analytical calculation provides an explanation for these discrepancies. Treating the base plate as a cantilever with projection $c = 98$ mm (from the column flange to the plate edge) yields $\sigma = 36.7$ MPa, consistent with the Idea StatiCa result. With projection $c = 48$ mm (from the column flange to the bolt) the result is $\sigma = 8.8$ MPa, consistent with the LIRA-SAPR result. Thus, the discrepancy between the software packages is attributable to different structural models of plate behavior, rather than to errors in the methods.

It should be noted that the ratio of plate thickness to plate dimension $t/L = 20/490 = 0.04$ corresponds to a thin plate, for which the use of shell elements in Idea StatiCa is physically more justified compared to the solid elements employed in LIRA-SAPR and SAFEM.

Conclusion. The paper presents a comparative analysis of the calculation of a 30K1 steel column base under a bending moment $M = 10$ kN·m using four approaches: SAFEM, Idea StatiCa Connection, LIRA-SAPR, and analytical calculation. It is established that each method is based on a different structural model of force transfer from the column to the plate, which significantly affects the obtained results.

Anchor bolt forces demonstrate satisfactory agreement among all methods. The discrepancy between Idea StatiCa Connection and LIRA-SAPR is 1.5% (13.9 and 13.69 kN respectively), and between Idea StatiCa and SAFEM – 14% (13.9 and 15.8 kN). The analytical value of 17.0 kN, obtained from the equilibrium condition $F = M/a = 10 \cdot 10^6/390 = 25.6$ kN for the bolt pair and 17.0 kN per single bolt, confirms the order of magnitude of the numerical results.

Base plate stresses are substantially dependent on the adopted computational model. Idea StatiCa Connection and LIRA-SAPR yield close results – 35.2 and 37.6 MPa respectively, with a discrepancy of 6%. The SAFEM value of 15.9 MPa is explained by the use of solid finite elements for a plate with a thickness-to-dimension ratio of $t/L = 20/490 = 0.04$, which classifies it as a thin plate. For such structures, the use of shell elements in Idea StatiCa is physically more justified.

The analytical calculation explains the mechanism behind the discrepancies between the methods. Treating the plate as a cantilever with projection $c = 98$ mm yields $\sigma = 36.7$ MPa; with projection $c = 48$ mm – $\sigma = 8.8$ MPa. The obtained analytical values confirm the order of magnitude of the numerical results and indicate that the discrepancy between the methods is attributable to different effective cantilever projections of the base plate, rather than to errors in the methods themselves.

The problem formulation in LIRA-SAPR and SAFEM through a pair of concentrated forces at the bolt nodes represents a simplification that does not reproduce the actual mechanism of force transfer through the column cross-section to the plate.

References

1. Aleshin, V., Bou Matar, O., & Van Den Abeele, K. (2020). On Applications of Semi-Analytical Methods of Contact Mechanics. *Frontiers in Mechanical Engineering*, *6*, Article 30. https://doi.org/10.3389/fmech.2020.00030
2. Altenbach, H., & Öchsner, A. (Eds.). (2020). Winkler Foundation Models. In *Encyclopedia of Continuum Mechanics*. Springer. https://doi.org/10.1007/978-3-662-55771-6
3. Bazhenov, V. A., Shkril', A. A., Maksymyuk, Yu. V., Martyniuk, I. Yu., & Maksymyuk, O. V. (2020). Semi-analytical method of finished elements in elastic and elastic-plastic position for curviline prismatic objects. *Opir materialiv i teoriia sporud (Resistance of Materials and Theory of Structures)*, (105), 24–32. https://doi.org/10.32347/2410-2547.2020.105.24-32
4. Bouzid, A. H., & Nechache, A. (2005). Flange leakage and bolt load in bolted flange joints. *Journal of Pressure Vessel Technology*, *127*(4), 413-418. https://doi.org/10.1115/1.2042477
5. EN 1993-1-8:2005. (2005). Eurocode 3: Design of steel structures - Part 1-8: Design of joints. Brussels: European Committee for Standardization.
6. IDEA StatiCa s.r.o. (2024). IDEA StatiCa Connection – Theoretical background. Retrieved from https://www.ideastatica.com/support-center/theoretical-background
7. Ivanchenko, H. M., Maksymiuk, Yu. V., Kozak, A. A., & Martyniuk, I. Yu. (2021). Construction of solving equations of the semi-analytical finite element method for prismatic bodies of complex shape. *Management of Development of Complex Systems*, (46), 55–62. https://doi.org/10.32347/2412-9933.2021.46.55-62
8. Kim, J., Yoon, J. C., & Kang, B. S. (2007). Finite element analysis and modeling of structure with bolted joints. *Applied Mathematical Modelling*, *31*(6), 895–911. https://doi.org/10.1016/j.apm.2006.03.020
9. LiraLand Group. (2023). LIRA-SAPR Software Package. Calculation and Design Examples: Educational manual. Kyiv: LiraLand. Retrieved from https://www.liraland.ua/download/private/lira/2023/lira_sapr_examples_ua.pdf
10. Mitsiuk, S. V., Maksym'yuk, Yu. V., & Mitsiuk, D. V. (2025). Comparative analysis of the results of flange connection calculation using the FEM and CFEM methods. *Modern Engineering and Innovative Technologies*, (40, Part 2), 88–96. https://doi.org/10.30890/2567-5273.2025-40-02
11. Piskunov, S. O., Mitsiuk, S. V., & Mitsiuk, D. V. (2026). Zonal Method for Contact Interaction Modeling in the Semi-Analytical Finite Element Method. *Resistance of Materials and Theory of Structures*, (116). (In press).
12. Stamatopoulos, G. N., & Ermopoulos, J. Ch. (2011). Experimental and analytical investigation of steel column bases. *Journal of Constructional Steel Research*, *67*(9), 1341–1357. https://doi.org/10.1016/j.jcsr.2011.03.007

13. Vabishchevych, M. O., & Storchak, D. A. (2022). Solution of nonlinear contact problems of deformation of nodal connections of steel structures. *Opir materialiv i teoriia sporud (Resistance of Materials and Theory of Structures)*, (108), 178–188. https://doi.org/10.32347/2410-2547.2022.108.178-188

АНОТАЦІЯ

Мета. Полягає в порівняльному аналізі результатів розрахунку бази сталевої колони під дією згинального моменту, отриманих за допомогою чотирьох підходів: напіваналітичного методу скінченних елементів (НМСЕ), програмного комплексу Idea StatiCa Connection, програмного комплексу LIRA-SAPR та аналітичного розрахунку, а також встановлення фізичних причин розбіжностей між методами.

Методика. Дослідження виконано на основі розрахунків бази колони перерізу 30К1 на плиті 490×490×20 мм при згинальному моменті M = 10 кН·м. В НМСЕ застосовано об'ємні елементи з контактною взаємодією через зональний метод, в Idea StatiCa — оболонкові елементи з моделлю Вінклера, в LIRA-SAPR — об'ємні елементи. Аналітичний розрахунок виконано для консольної пластини з трикутним епюром тиску фундаменту.

Результати. Встановлено задовільну збіжність зусиль в анкерних болтах між усіма методами — розбіжність не перевищує 14%. Виявлено суттєву залежність напружень в опорній плиті від прийнятої розрахункової схеми: Idea StatiCa та LIRA-SAPR дають близькі результати – 35.2 та 37.6 МПа при розбіжності 6%, тоді як НМСЕ дає 15.9 МПа через застосування об'ємних скінченних елементів для тонкої пластини (t/L = 0.04). Аналітичний розрахунок пояснює отримані розбіжності різним розрахунковим вильотом консолі плити.

Наукова новизна. Вперше виконано комплексний порівняльний аналіз чотирьох методів розрахунку бази колони. Встановлено що зусилля в болтах демонструють високу збіжність між методами, тоді як напруження в плиті залежать від типу скінченних елементів – різниця досягає 2.4 рази. Показано що для тонкої пластини оболонкові елементи є фізично більш обґрунтованими, що підтверджується аналітичним розрахунком.

Практична значимість. Отримані результати дозволяють інженеру-проектувальнику здійснити обґрунтований вибір розрахункового підходу при проектуванні баз сталевих колон з урахуванням особливостей кожного методу. Встановлені закономірності сприяють вдосконаленню методик розрахунку вузлів кріплення колон до фундаменту та підвищенню точності оцінки їх напружено-деформованого стану.

Ключові слова: *база колони, напружено-деформований стан, напіваналітичний метод скінченних елементів (НМСЕ), Idea StatiCa Connection, LIRA-SAPR, згинальний момент, консольна пластина.*

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